GCSE Maths Grade 9 Paper 1 Revision Warm-up: Written Solutions

Question 1

Solve the simultaneous equations $0.3x + 0.2y = 2.1$ and $x - y = 4$.

Solution

From $x - y = 4$, $x = y + 4$.
Substitute into $0.3x + 0.2y = 2.1$:
$0.3(y + 4) + 0.2y = 2.1,$ so $0.5y + 1.2 = 2.1.$
$y = 1.8,$ and $x = y + 4 = 5.8.$
Final answer: $x = 5.8, \ y = 1.8$

Question 2

$(x + a)(x - 3) + b \equiv x^2 + 2x - 11$.
Find a and b.

Solution

Expanding gives $(x + a)(x - 3) + b = x^2 + (a - 3)x - 3a + b$.
Compare coefficients with $x^2 + 2x - 11$.
$a - 3 = 2,$ so $a = 5.$
$-3a + b = -11,$ so $-15 + b = -11$ and $b = 4$.
Final answer: $a = 5, \ b = 4$

Question 3

Rationalise and simplify $\frac{6}{\sqrt{5}-\sqrt{2}}$.

Solution

Multiply the numerator and denominator by $\sqrt{5} + \sqrt{2}$.
The denominator becomes $5 - 2 = 3$.
So $\frac{6}{\sqrt{5}-\sqrt{2}} = \frac{6\sqrt{5}+6\sqrt{2}}{3} = 2\sqrt{5} + 2\sqrt{2}$.
Final answer: $2\sqrt{5} + 2\sqrt{2}$

Question 4

The line $3x + 4y = 24$ crosses the axes at A and B.
Work out the area of triangle OAB.

Solution

For $3x + 4y = 24$, the x-intercept is 8 and the y-intercept is 6.
Triangle OAB is right-angled at O.
Area $= \frac{1}{2} \times 8 \times 6 = 24$.
Final answer: 24

Question 5

A bag has red and blue counters.
$P(\textbf{red}) = \frac{1}{2}$**.**
Two counters are chosen without replacement and $P(\textbf{both red}) = \frac{2}{9}$**. How many counters were in the bag at first?**

Solution

Let there be n red counters and n blue counters, since $P(\text{red}) = \frac{1}{2}$.

The probability of both red is $\frac{n}{2n} \times \frac{n-1}{2n-1} = \frac{2}{9}$.

So $(\frac{1}{2})\frac{n-1}{2n-1} = \frac{2}{9}$.

$9n - 9 = 8n - 4,$ so $n = 5$.

Total counters $= 10$.

Final answer: 10

Question 6

Evaluate $\frac{(3.6\times10^5)(4\times10^{-3})}{8\times10^2}$**.**
Give your answer in standard form.

Solution

$(3.6 \times 10^5)(4 \times 10^{-3}) = 14.4 \times 10^2$.

Dividing by 8×10^2 gives $(\frac{14.4}{8}) \times 10^0$.

This is 1.8×10^0.

Final answer: 1.8×10^0

Question 7

A rectangle has length 8.0 **cm and width** 2.0 **cm, each to 1 decimal place.**
Find the upper bound for its perimeter.

Solution

The upper bound for the length is 8.05 cm.

The upper bound for the width is 2.05 cm.

Upper bound for perimeter $= 2(8.05 + 2.05) = 20.2\,\text{cm}$.

Final answer: 20.2 cm

Question 8

Work out the coordinates of the turning point of $y = x^2 - 1.2x + 0.11$**.**

Solution

$x^2 - 1.2x + 0.11 = (x - 0.6)^2 - 0.36 + 0.11$.

This is $(x - 0.6)^2 - 0.25$.

The turning point is (0.6, -0.25).

Final answer: $(0.6, -0.25)$

Question 9

A metal has density 7.8 g/cm^3.
A cuboid of the metal measures 2.5 cm by 4 cm by 0.6 cm.
Find its mass, giving your answer in grams.

Solution

Volume $= 2.5 \times 4 \times 0.6 = 6\,\text{cm}^3$.

Mass $=$ density $\times$ volume $= 7.8 \times 6 = 46.8\,\text{g}$.

Final answer: 46.8 g

Question 10

The line $y = mx + 5$ is tangent to the circle $x^2 + y^2 = 20$.
Find the possible values of m.

Solution

Substitute $y = mx + 5$ into $x^2 + y^2 = 20$.

$x^2 + (mx + 5)^2 = 20$ gives $(m^2 + 1)x^2 + 10mx + 5 = 0$.

For a tangent, the discriminant is 0:

$(10m)^2 - 4(m^2 + 1)(5) = 0$.

$80m^2 - 20 = 0,$ so $m^2 = \frac{1}{4}$.

$m = \frac{1}{2}$ or $m = -\frac{1}{2}$.

Final answer: $m = \frac{1}{2}$ or $m = -\frac{1}{2}$

Question 11

For events F and S, $P(F) = \frac{3}{5}$, $P(S) = \frac{8}{15}$, and $P(F \cup S) = \frac{5}{6}$.
Find $P(F \cap S')$.

Solution

$P(F \cap S) = P(F) + P(S) - P(F \cup S)$.

$P(F \cap S) = \frac{3}{5} + \frac{8}{15} - \frac{5}{6} = \frac{3}{10}$.

$P(F \cap S') = P(F) - P(F \cap S) = \frac{3}{5} - \frac{3}{10} = \frac{3}{10}$.

Final answer: $\frac{3}{10}$

Question 12

On a histogram, $0 < x \leq 4$ has frequency density 2.5, $4 < x \leq 10$ has frequency 9, and the total frequency for $0 < x \leq 14$ is 31.
Find the frequency density for $10 < x \leq 14$.

Solution

Frequency for $0 < x \leq 4$ is $4 \times 2.5 = 10$.

Frequency for $4 < x \leq 10$ is 9.

The remaining frequency is $31 - 10 - 9 = 12$.

The class width from 10 to 14 is 4, so density $= \frac{12}{4} = 3$.

Final answer: 3

Question 13

In a group, 40% of the people are boys.
25% of the boys and 50% of the girls study music.
There are 18 more girls than boys who study music.
Work out the number of people in the group.

Solution

Let the total number of people be N.

Boys who study music = 25% of 40% of $N = 0.1N$.

Girls who study music = 50% of 60% of $N = 0.3N$.

The difference is $0.2N = 18$, so $N = 90$.

Final answer: 90

Question 14

The points $A(-2, 5)$, $B(10, -1)$ and $C(k, 2)$ are collinear.
Find k.

Solution

The gradient of AB is $(-1 - 5)/(10 - (-2)) = -\frac{6}{12} = -\frac{1}{2}$.
Since C is collinear, the gradient of AC is also $-\frac{1}{2}$.
$(2 - 5)/(k - (-2)) = -\frac{1}{2}$.
$-\frac{3}{k+2} = -\frac{1}{2}$, so $k + 2 = 6$ and $k = 4$.
Final answer: $k = 4$

Question 15

Solve $y = x + 3$ and $xy = 10$.

Solution

Substitute $y = x + 3$ into $xy = 10$.

$x(x + 3) = 10$, so $x^2 + 3x - 10 = 0$.

$(x - 2)(x + 5) = 0$, so $x = 2$ or $x = -5$.

The corresponding y-values are 5 and -2.

Final answer: $(2, 5)$ and $(-5, -2)$

Question 16

The gradient of the line joining $A(2, k)$ and $B(k, 8)$ is -2.
Find k.

Solution

Gradient $AB = \frac{8-k}{k-2}$.
Set this equal to -2:
$\frac{8-k}{k-2} = -2$.
$8 - k = -2k + 4$, so $k = -4$.
Final answer: $k = -4$

Question 17

A polygon has one exterior angle of $72°$. Each of its other exterior angles is $24°$.
Work out the number of sides of the polygon.

Solution

The exterior angles of any polygon sum to $360°$.

$72 + 24(n-1) = 360.$

$24(n-1) = 288,$ so $n - 1 = 12.$

$n = 13.$

Final answer: 13

Question 18

An iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{18}{x_n}\right)$ converges to a positive fixed point x; find x, giving your answer in exact form.

Solution

At the fixed point, $x = \frac{1}{2}(x + \frac{18}{x})$.

$2x = x + \frac{18}{x},$ so $x = \frac{18}{x}.$

$x^2 = 18.$

The positive fixed point is $x = 3\sqrt{2}$.

Final answer: $3\sqrt{2}$

Question 19

$f(x) = x^2 - 3x$.
Given that $f(a) = f(a+4)$, work out the value of a.

Solution

$f(a) = a^2 - 3a.$

$f(a+4) = (a+4)^2 - 3(a+4) = a^2 + 5a + 4.$

Equating gives $a^2 - 3a = a^2 + 5a + 4.$

$-8a = 4,$ so $a = -\frac{1}{2}.$

Final answer: $a = -\frac{1}{2}$

Question 20

The equation $x^2 + kx + 25 = 0$ has exactly one solution.
Find all positive values of k.

Solution

A quadratic has exactly one solution when its discriminant is 0.

For $x^2 + kx + 25 = 0$, $k^2 - 4(1)(25) = 0.$

$k^2 = 100,$ so $k = 10$ or $k = -10.$

The positive value is $k = 10$.

Final answer: $k = 10$

Question 21

Point P lies on AB, where $A(-2, 5)$, $B(10, -1)$, and $AP : PB = 1 : 3$.
Find the coordinates of P.

Solution

P divides AB in the ratio 1 : 3, so P is one quarter of the way from A to B.

$B - A = (12, -6)$.

One quarter of this is (3, -1.5).

$P = (-2, 5) + (3, -1.5) = (1, 3.5)$.

Final answer: $(1, 3.5)$

Question 22

After a 12.5% decrease, a price is £70.
Find the original price.

Solution

After a 12.5% decrease, the multiplier is $0.875 = \frac{7}{8}$.

Original price $\times \dfrac{7}{8} = £70$.

Original price $= £70 \times \dfrac{8}{7} = £80$.

Final answer: £80

Question 23

A geometric sequence has $u_2 = 1.2$ and $u_5 = 32.4$.
The common ratio is positive.
Find u_4.

Solution

$\dfrac{u_5}{u_2} = \dfrac{32.4}{1.2} = 27$.

This is r^3, so the positive common ratio is $r = 3$.

$u_4 = u_2 \times r^2 = 1.2 \times 9 = 10.8$.

Final answer: 10.8

Question 24

The mean of n numbers is 6.4.
When 10.9 is added, the mean becomes 6.7.
Work out the value of n.

Solution

The original total is $6.4n$.

After adding 10.9, $\frac{6.4n+10.9}{n+1} = 6.7$.

$6.4n + 10.9 = 6.7n + 6.7$.

$4.2 = 0.3n,$ so $n = 14$.

Final answer: 14

Question 25

Solve $3^{x+1} + 3^x = 36$.

Solution

$3^{x+1} + 3^x = 3 \times 3^x + 3^x = 4 \times 3^x$.

$4 \times 3^x = 36,$ so $3^x = 9$.

Therefore $x = 2$.

Final answer: $x = 2$

Question 26

Triangle ABC has $AB = 6$ cm, $AC = 12$ cm and $\angle ABC = 90°$.
Work out the area of triangle ABC, giving your answer in exact form.

Solution

Since angle ABC is $90°$, AC is the hypotenuse.

$BC^2 = 12^2 - 6^2 = 108,$ so $BC = 6\sqrt{3}$.

Area $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}$.

Final answer: $18\sqrt{3}$

Question 27

Two similar solids have surface areas in the ratio $9 : 16$.
The larger volume is 296 cm^3 greater than the smaller volume.
Find the larger volume.

Solution

Surface area ratio $9 : 16$ gives linear scale factor ratio $3 : 4$.

Volume ratio is therefore $3^3 : 4^3 = 27 : 64$.

The difference is $64 - 27 = 37$ parts .

37 parts $= 296$ cm^3, so 1 part $= 8$ cm^3.

The larger volume is 64 parts $= 512$ cm^3.

Final answer: 512 cm^3

Question 28

Solve $x^2 - 0.7x + 0.1 < 0$.

Solution

Factor $x^2 - 0.7x + 0.1$ as $(x - 0.2)(x - 0.5)$.

The parabola opens upwards.

It is below 0 between the roots, so $0.2 < x < 0.5$.

Final answer: $0.2 < x < 0.5$

Question 29

Triangle ABC has $\angle BAC = 30°$, $\angle ABC = 45°$ and $BC = 4$.
Find AC, giving your answer in exact form.

Solution

BC is opposite angle BAC, so BC is side a.

AC is opposite angle ABC, so AC is side b.

By the sine rule, $\dfrac{b}{\sin 45°} = \dfrac{4}{\sin 30°}$.

$b = 4 \times \dfrac{\sin 45°}{\sin 30°} = 4\sqrt{2}$.

Final answer: $4\sqrt{2}$

Question 30

Triangle ABC has $AB = 3.6$ cm, $BC = 5$ cm and $\angle ABC = 30°$.
Find the area of the triangle.

Solution

Area $= \dfrac{1}{2} \times AB \times BC \times \sin\angle ABC$.

Area $= \dfrac{1}{2} \times 3.6 \times 5 \times \sin 30°$.

Area $= \frac{1}{2} \times 3.6 \times 5 \times \frac{1}{2} = 4.5\,\text{cm}^2$.

Final answer: 4.5 cm^2

Question 31

A set of data has lower quartile 3.2 and interquartile range 4.6.
Every value is multiplied by 2.
Find the new upper quartile.

Solution

Original upper quartile = lower quartile + interquartile range $= 3.2 + 4.6 = 7.8$.

Multiplying every value by 2 also multiplies each quartile by 2.

New upper quartile $= 15.6$.

Final answer: 15.6

Question 32

The area of a semicircle is 50π.
Work out the perimeter of the semicircle, giving your answer in exact form.

Solution

For the semicircle, $\frac{1}{2}\pi r^2 = 50\pi$.

$r^2 = 100,$ so $r = 10$.

Perimeter = half the circumference + diameter $= 10\pi + 20$.

Final answer: $10\pi + 20$

Question 33

Triangle ABC has $AB = 7$ cm, $BC = 9$ cm and $\angle ABC = 60°$.
Find AC, giving your answer in exact form.

Solution

Use the cosine rule.

$AC^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \times \cos 60°$.

$AC^2 = 49 + 81 - 63 = 67$.

$AC = \sqrt{67}$ cm.

Final answer: $\sqrt{67}$ cm

Question 34

A line segment has endpoints $A(-3,4)$ and $B(5,0)$.
It is enlarged by scale factor $-\frac{1}{2}$, centre $(0,0)$.
Find the image of the midpoint of AB.

Solution

The midpoint of AB is $\left(\dfrac{-3+5}{2}, \dfrac{4+0}{2}\right) = (1,2)$.

Enlarging from the origin by scale factor $-\dfrac{1}{2}$ multiplies both coordinates by $-\dfrac{1}{2}$.

The image is $\left(-\dfrac{1}{2}, -1\right)$.

Final answer: $\left(-\frac{1}{2}, -1\right)$

Question 35

Solve $\frac{2}{x-1} + \frac{3}{x+1} = 4$, giving your answers in the form $\frac{a \pm \sqrt{b}}{c}$.

Solution

Multiply by $(x-1)(x+1)$:

$2(x+1) + 3(x-1) = 4(x^2-1)$.

$5x - 1 = 4x^2 - 4, \text{ so } 4x^2 - 5x - 3 = 0$.

Using the quadratic formula gives $x = \dfrac{5 \pm \sqrt{73}}{8}$.

Final answer: $\frac{5 \pm \sqrt{73}}{8}$

Question 36

A price is increased by 20% then decreased by 20%.
The final price is £144.
Find the original price.

Solution

The combined percentage multiplier is $1.2 \times 0.8 = 0.96$.

Original price $\times 0.96 = £144$.

Original price $= £144 \div 0.96 = £150$.

Final answer: £150

Question 37

A sector has radius 6 cm.
Its area is equal to the area of a semicircle with radius 4 cm.
Find the angle of the sector.

Solution

The area of the semicircle is $(\frac{1}{2})\pi \times 4^2 = 8\pi$.

The sector area is also 8π.

Sector area $= (\theta/360) \times \pi \times 6^2$.

So $(\theta/360) \times 36\pi = 8\pi$.

$\theta/10 = 8, \text{ so } \theta = 80°$.

Final answer: $80°$

Question 38

For $f(x) = \frac{x+1}{x-2}$, find $f(f(x))$.

Solution

$f(f(x)) = (\frac{x+1}{x-2} + 1)/(\frac{x+1}{x-2} - 2)$.

The numerator is $\frac{x+1+x-2}{x-2} = \frac{2x-1}{x-2}$.

The denominator is $\frac{x+1-2x+4}{x-2} = \frac{5-x}{x-2}$.

Dividing gives $\frac{2x-1}{5-x}$.

Final answer: $\frac{2x-1}{5-x}$

Question 39

A pressure of 2.4 N/cm^2 acts on an area of 0.75 cm^2.
Find the force.

Solution

Pressure = force/area.

Force = pressure × area = 2.4 × 0.75 = 1.8 N.

Final answer: 1.8 N

Question 40

y is proportional to $\frac{x^2}{z}$.
When $x = 1.5$, $z = 0.6$, $y = 7.5$.
Find y when $x = 0.9$ and $z = 0.3$.

Solution

$y = kx^2/z$.

Using $x = 1.5$, z = 0.6, $y = 7.5$:

$7.5 = k \times 1.5^2/0.6,$ so $k = 2$.

When $x = 0.9$ and $z = 0.3$, $y = 2 \times 0.9^2/0.3 = 5.4$.

Final answer: 5.4

Question 41

A quadratic graph has roots 3 and -5, and passes through $(1, -24)$.
Find its equation.

Solution

With roots 3 and -5, the equation is $y = a(x - 3)(x + 5)$.

It passes through (1, -24), so $-24 = a(1 - 3)(1 + 5)$.

$-24 = -12a,$ so $a = 2$.

The equation is $y = 2(x - 3)(x + 5)$.

Final answer: $y = 2(x - 3)(x + 5)$

Question 42

The roots of $x^2 + px + q = 0$ are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.
Find p and q.

Solution

The sum of the roots is $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$.
For $x^2 + px + q = 0$, the sum of roots is -p, so $p = -4$.
The product is $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$, so $q = 1$.
Final answer: $p = -4,\ q = 1$

Question 43

The difference between the squares of two consecutive positive odd integers is 64.
Find the integers.

Solution

Let the smaller odd integer be n, so the next one is $n + 2$.
$(n + 2)^2 - n^2 = 64$.
$4n + 4 = 64,\ \text{so}\ n = 15$.
The integers are 15 and 17.
Final answer: 15 and 17

Question 44

The equation $x^2 + px + 12 = 0$ has integer roots.
The difference between the roots is 1.
Find the possible values of p.

Solution

Integer roots with product 12 and difference 1 are 3 and 4, or -3 and -4.
For roots 3 and 4, the sum is 7, so $p = -7$.
For roots -3 and -4, the sum is -7, so $p = 7$.
Therefore $p = 7$ or $p = -7$.
Final answer: $p = 7$ or $p = -7$

Question 45

Given $a : b = 2 : 5$ and $b : c = 3 : 4$, and $a + c = 78$, work out b.

Solution

Make the b-parts equal.
$a : b = 2 : 5$ becomes $6 : 15$.
$b : c = 3 : 4$ becomes $15 : 20$.
So $a : b : c = 6 : 15 : 20$.
$a + c = 26$ parts $= 78$, so 1 part $= 3$.
$b = 15$ parts $= 45$.
Final answer: 45

Question 46

A mass m is 5.0 kg to 1 decimal place.
Give the error interval for $\frac{1}{m}$, using fractions.

Solution

Since m is 5.0 kg to 1 decimal place, $4.95 \leq m < 5.05$.

Because $\dfrac{1}{m}$ decreases as m increases, reverse the inequalities:

$\frac{1}{5.05} < \frac{1}{m} \leq \frac{1}{4.95}$.

This gives $\frac{20}{101} < \frac{1}{m} \leq \frac{20}{99}$.

Final answer: $\frac{20}{101} < \frac{1}{m} \leq \frac{20}{99}$

Question 47

If $\tan\theta = \frac{3}{4}$, find $\sin\theta$ for acute θ.

Solution

$\tan\theta = \dfrac{3}{4}$, so the opposite and adjacent sides can be 3 and 4.

The hypotenuse is 5.

Therefore $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{5}$.

Final answer: $\frac{3}{5}$

Question 48

Given $f(x) = \frac{2x-1}{x+3}$, find $f^{-1}(x)$, giving your answer as a single fraction.

Solution

Let $y = \frac{2x-1}{x+3}$.

$y(x+3) = 2x - 1$.

$yx + 3y = 2x - 1$.

$x(y-2) = -1 - 3y$.

$x = \frac{3y+1}{2-y}$.

Therefore $f^{-1}(x) = \frac{3x+1}{2-x}$.

Final answer: $\frac{3x+1}{2-x}$

Question 49

Find n if $8^{n-1} = 32^{2n+1}$.

Solution

Write both sides as powers of 2.

$8^{n-1} = 2^{3n-3}$.

$32^{2n+1} = 2^{10n+5}$.

So $3n - 3 = 10n + 5$.

$-8 = 7n,$ so $n = -\frac{8}{7}$.

Final answer: $n = -\frac{8}{7}$

Question 50

Make x the subject of $y = \frac{x+a}{x-a}$.

Solution

$y = \frac{x+a}{x-a}$.

$y(x-a) = x + a$.

yx $-ay = x + a$.

$x(y-1) = a(y+1)$.

$x = a\frac{y+1}{y-1}$.

Final answer: $x = \frac{a(y+1)}{y-1}$

Question 51

In a capture-recapture estimate, t fish are tagged.
Later, 48 fish are caught and 6 are tagged.
The estimated population is 210 more than t.
Find t.

Solution

Capture-recapture estimate: population $\approx$ tagged first $\times$ caught later / tagged later.

The estimate is $t \times 48 \div 6 = 8t$.

The estimate is also t + 210.

$8t = t + 210,$ so $7t = 210$.

$t = 30$.

Final answer: 30

Question 52

Solve the simultaneous equations $x^2 + y^2 = 25$ and $y = 2x - 5$.

Solution

Substitute $y = 2x - 5$ into $x^2 + y^2 = 25$.

$x^2 + (2x-5)^2 = 25$.

$5x^2 - 20x = 0,$ so $5x(x-4) = 0$.

$x = 0$ or $x = 4$.

The solutions are (0, -5) and (4, 3).

Final answer: $(0, -5)$ and $(4, 3)$

Question 53

A sequence has $u_{n+2} = u_{n+1} + u_n$, with $u_3 = 7$ and $u_4 = 11$.
Find u_1.

Solution

Work backwards from $u_4 = 11$ and $u_3 = 7$.

$u_4 = u_3 + u_2$, so $11 = 7 + u_2$ and $u_2 = 4$.

$u_3 = u_2 + u_1$, so $7 = 4 + u_1$ and $u_1 = 3$.

Final answer: 3

Question 54

A line passes through $(6, -1)$ and is perpendicular to $3x - 2y = 8$.
Work out the y-intercept of the line.

Solution

$3x - 2y = 8$ rearranges to $y = \dfrac{3}{2}x - 4$, so its gradient is $\dfrac{3}{2}$.

A perpendicular line has gradient $-\dfrac{2}{3}$.

Through $(6,\ -1) : y = (-\frac{2}{3})x + c$.

$-1 = -4 + c,\ $ so $c = 3$.

Final answer: 3

Question 55

A straight line makes an angle of $150°$ with the positive x-axis.
Find its gradient in exact form.

Solution

The gradient of a line making angle θ with the positive x-axis is $\tan\theta$.

$\tan 150° = -\tan 30° = -\dfrac{1}{\sqrt{3}}$.

In rationalised form this is $-\dfrac{\sqrt{3}}{3}$.

Final answer: $-\frac{\sqrt{3}}{3}$

Question 56

A sector has arc length 5π cm and perimeter $26 + 5\pi$ cm; find its area, giving your answer in terms of π.

Solution

Perimeter $= 2r+$ arc length.

$26 + 5\pi = 2r + 5\pi,\ $ so $r = 13$.

Sector area $= \dfrac{1}{2} \times$ radius $\times$ arc length.

Area $= \frac{1}{2} \times 13 \times 5\pi = 65\pi/2\,\text{cm}^2$.

Final answer: $\frac{65\pi}{2}$ cm^2

Question 57

A sequence has $u_n = an^2 + bn$.
Given $u_3 = 21$ and $u_5 = 55$, find u_4.

Solution

$u_n = an^2 + bn$.

$u_3 = 21$ gives $9a + 3b = 21$, so $3a + b = 7$.

$u_5 = 55$ gives $25a + 5b = 55$, so $5a + b = 11$.

Subtracting gives $2a = 4$, so $a = 2$ and $b = 1$.

$u_4 = 2(4^2) + 4 = 36$.

Final answer: 36

Question 58

Solve $2\cos x = 1$ for $0° \le x \le 360°$.

Solution

$2\cos x = 1$ gives $\cos x = \dfrac{1}{2}$.

In $0° \le x \le 360°$, this happens in quadrants 1 and 4.

$x = 60°$ or $x = 300°$.

Final answer: $x = 60°$ or $x = 300°$

Question 59

In a bag of counters, $R : B = 3 : 5$ and $B : G = 2 : 1$.
There are 33 counters that are not blue.
Work out the total number of counters.

Solution

Make the B-parts equal.

$R : B = 3 : 5$ becomes $6 : 10$.

$B : G = 2 : 1$ becomes $10 : 5$.

So $R : B : G = 6 : 10 : 5$.

Not blue means $R + G = 11$ parts $= 33$, so 1 part $= 3$.

Total $= 21$ parts $= 63$.

Final answer: 63

Question 60

The HCF of $2^a \cdot 3^4 \cdot 5$ and $2^3 \cdot 3^b \cdot 5^2$ is $2^2 \cdot 3^2 \cdot 5$.
Find the least possible value of $a + b$.

Solution

For the HCF, take the smaller power of each prime.

min(a, 3) = 2, so the least possible value of a is 2.

min(4, b) = 2, so the least possible value of b is 2.

Therefore the least possible value of $a + b$ is 4.

Final answer: 4

Question 61

Write $0.\dot{1}2\dot{3}$ as a fraction in its simplest form.

Solution

Let $x = 0.123123...$

Then $1000x = 123.123123...$

$1000x - x = 123,$ so $999x = 123.$

$x = \frac{123}{999} = \frac{41}{333}$.

Final answer: $\frac{41}{333}$

Question 62

A, B, C, D are in order on a circle.
If $\angle ABC = 112°$ and $\angle ACD = 34°$, find $\angle ADC$.

Solution

Opposite angles in a cyclic quadrilateral add to 180°.

Angle ABC and angle ADC are opposite.

Angle $ADC = 180° - 112° = 68°$.

Final answer: 68°

Question 63

Simplify $\frac{2}{x+1} - \frac{3}{x-2}$, giving your answer as a single fraction.

Solution

Use the common denominator $(x + 1)(x - 2)$.

$\frac{2}{x+1} - \frac{3}{x-2} = [2(x - 2) - 3(x + 1)]/[(x + 1)(x - 2)]$.

The numerator is $2x - 4 - 3x - 3 = -x - 7$.

The denominator is $x^2 - x - 2$.

Final answer: $\frac{-x-7}{x^2-x-2}$

Question 64

Find the coefficient of x in $(x - 2)(x + 3)(2x - 1)$.

Solution

First expand $(x - 2)(x + 3) = x^2 + x - 6$.

Then $(x^2 + x - 6)(2x - 1) = 2x^3 + x^2 - 13x + 6$.

The coefficient of x is -13.

Final answer: -13

Question 65

A cube has volume 0.000064 m^3.
Find its side length in cm.

Solution

$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$.

$0.000064 \text{ m}^3 = 64 \text{ cm}^3$.

If the cube has side length s, then $s^3 = 64$.

$s = 4\,\text{cm}$.

Final answer: 4 cm

Question 66

A 3-digit code has a prime digit, then a factor of 12, then a square number digit. Repetition is allowed. How many codes are possible?

Solution

Prime digit choices: 2, 3, 5, 7, so 4 choices.

Factor of $12 digit choices$: 1, 2, 3, 4, 6, so 5 choices.

Square number digit choices: 0, 1, 4, 9, so 4 choices.

Total codes $= 4 \times 5 \times 4 = 80$.

Final answer: 80

Question 67

A cone and a sphere have the same radius r.
The cone has height $6r$.
Find the ratio of the volume of the cone to the volume of the sphere.

Solution

Cone volume $= (\frac{1}{3})\pi r^2(6r) = 2\pi r^3$.
Sphere volume $= (\frac{4}{3})\pi r^3$.
Ratio cone : sphere $= 2 : \dfrac{4}{3} = 6 : 4 = 3 : 2$.
Final answer: $3 : 2$

Question 68

The 2nd term of a geometric sequence is $2 + \sqrt{3}$.
The 3rd term is $9 + 5\sqrt{3}$.
Work out the 4th term, giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

Solution

Common ratio $= \frac{9+5\sqrt{3}}{2+\sqrt{3}}$.
Rationalising gives $(9 + 5\sqrt{3})(2 - \sqrt{3}) = 3 + \sqrt{3}$.
The 4th term is $(9 + 5\sqrt{3})(3 + \sqrt{3})$.
This equals $42 + 24\sqrt{3}$.
Final answer: $42 + 24\sqrt{3}$

Question 69

$\overrightarrow{PQ} = 2a - b$, $\overrightarrow{QR} = a + 3b$, and $\overrightarrow{PS} = ka + 5b$.
Find k so that P, R, S are collinear.

Solution

$PR = PQ + QR = (2a - b) + (a + 3b) = 3a + 2b$.
For P, R, S to be collinear, PS must be a multiple of PR.
$ka + 5b = \lambda(3a + 2b)$.
From the b coefficient, $5 = 2\lambda$, so $\lambda = \frac{5}{2}$.
$k = 3\lambda = \frac{15}{2}$.
Final answer: $k = \frac{15}{2}$

Question 70

A rectangle has sides $x + 2$ and $x - 1$.
Its area is $2x^2 - 5$.
Find x, giving your answer in the form $\frac{a+\sqrt{b}}{2}$.

Solution

$(x + 2)(x - 1) = x^2 + x - 2$.
Set this equal to $2x^2 - 5$:
$x^2 + x - 2 = 2x^2 - 5$.
$x^2 - x - 3 = 0$.
$x = \dfrac{1 \pm \sqrt{13}}{2}$. Since the side $x - 1$ must be positive, $x = \dfrac{1 + \sqrt{13}}{2}$.
Final answer: $x = \frac{1+\sqrt{13}}{2}$

Question 71

For integer n, simplify $(n+2)^3 - (n-2)^3$, giving your answer in the form $an^2 + b$.

Solution

$(n+2)^3 = n^3 + 6n^2 + 12n + 8.$

$(n-2)^3 = n^3 - 6n^2 + 12n - 8.$

Subtracting gives $12n^2 + 16$.

Final answer: $12n^2 + 16$

Question 72

$\overrightarrow{OA} = 2a + 3b$ and $\overrightarrow{OB} = 8a + 15b$.
Point P lies on AB with $AP : PB = 1 : 2$.
Find $\overrightarrow{OP}$.

Solution

P is one third of the way from A to B.

$OP = OA + \frac{1}{3}(OB - OA).$

This is $(\frac{2}{3})OA + (\frac{1}{3})OB.$

$OP = [2(2a + 3b) + (8a + 15b)]/3 = \frac{12a+21b}{3} = 4a + 7b.$

Final answer: $4a + 7b$

Question 73

A quadratic sequence begins $5,\ 11,\ 21,\ 35, \ldots$. Which term of the sequence is 203?

Solution

The first differences are 6, 10, 14, so the second difference is 4.

The nth term has leading term $2n^2$.

Using the first term gives $2(1)^2 + c = 5$, so $c = 3$.

The nth term is $2n^2 + 3$.

$2n^2 + 3 = 203,$ so $n^2 = 100$ and $n = 10$.

Final answer: The 10th term

Question 74

After two successive 10% increases, a price is £60.50.
Find the original price.

Solution

Two 10% increases give multiplier $1.1 \times 1.1 = 1.21$.

Original price $\times 1.21 = £60.50$.

Original price $= £60.50 \div 1.21 = £50$.

Final answer: £50

Question 75

A cuboid has side lengths 2.4 cm, 3.2 cm and h cm.
The longest diagonal of the cuboid is 5 cm.
Find h.

Solution

For a cuboid, $d^2 = 2.4^2 + 3.2^2 + h^2$.

$5^2 = 5.76 + 10.24 + h^2$.

$25 = 16 + h^2,$ so $h^2 = 9$.

$h = 3\,\text{cm}$.

Final answer: 3 cm

Question 76

A sector has arc length 3π cm and area 18π cm^2.
Find its radius.

Solution

Sector area $= \dfrac{1}{2} \times \text{radius} \times \text{arc length}$.

$18\pi = \frac{1}{2} \times r \times 3\pi$.

$18\pi = \dfrac{3\pi r}{2}$, so $r = 12$ cm.

Final answer: 12 cm

Question 77

Simplify $\frac{(27x^6)^{2/3}}{3x}$.

Solution

$(27x^6)^{\frac{2}{3}} = 27^{\frac{2}{3}} \times (x^6)^{\frac{2}{3}}$.

$27^{\frac{2}{3}} = 9$ and $(x^6)^{\frac{2}{3}} = x^4$.

So the numerator is $9x^4$.

$9x^4/(3x) = 3x^3$.

Final answer: $3x^3$

Question 78

A biased coin has $P(H) = 0.6$.
It is tossed twice.
Given that at least one head is thrown, find $P(\text{exactly one head})$.

Solution

$P(\text{at least one head}) = 1 - P(\text{TT}) = 1 - 0.4^2 = 0.84$.

$P(\text{exactly one head}) = HT + TH = 0.6 \times 0.4 + 0.4 \times 0.6 = 0.48$.

Conditional probability $= \frac{0.48}{0.84} = \frac{4}{7}$.

Final answer: $\frac{4}{7}$

Question 79

Write $0.\dot{2}\dot{7} + 0.\dot{3}$ as a fraction in its simplest form.

Solution

$0.272727... = \frac{27}{99} = \frac{3}{11}$.
$0.333333... = \frac{1}{3}$.
$\frac{3}{11} + \frac{1}{3} = \frac{9}{33} + \frac{11}{33} = \frac{20}{33}$.
Final answer: $\frac{20}{33}$

Question 80

From A, B is 8 km away on a bearing of 060°; C is due east of A and due south of B; find AC, giving your answer in exact form.

Solution

From A to B, the east component is $8\sin 60° = 4\sqrt{3}$.

Since C is due east of A and due south of B, AC is this east component.

$AC = 4\sqrt{3}$ km.

Final answer: $4\sqrt{3}$ km

Question 81

Circle C has centre $(0, 0)$ and radius 13.
Point P lies on C and has x-coordinate 5.
Find the possible equations of the tangent to C at P.

Solution

Since P lies on $x^2 + y^2 = 169$ and has x-coordinate 5, $y^2 = 169 - 25 = 144$.

So $y = 12$ or $y = -12$.

For $P(5, 12)$, the tangent is $5x + 12y = 169$.

For $P(5, -12)$, the tangent is $5x - 12y = 169$.

Final answer: $5x + 12y = 169$ or $5x - 12y = 169$

Question 82

Simplify $(2 + \sqrt{5})^2 - (2 - \sqrt{5})^2$.

Solution

Use the difference of two squares.
$(2 + \sqrt{5})^2 - (2 - \sqrt{5})^2 = [(2 + \sqrt{5}) - (2 - \sqrt{5})][(2 + \sqrt{5}) + (2 - \sqrt{5})]$.
This is $(2\sqrt{5})(4) = 8\sqrt{5}$.
Final answer: $8\sqrt{5}$

Question 83

A tangent to $x^2 + y^2 = 25$ has gradient $-\frac{3}{4}$.
Find the possible y-coordinates of the point where the tangent touches the circle.

Solution

The tangent gradient is $-\frac{3}{4}$, so the radius gradient is $\frac{4}{3}$.

For a point (x, y) on the circle, $\dfrac{y}{x} = \dfrac{4}{3}$, so $y = \dfrac{4x}{3}$.

Substitute into $x^2 + y^2 = 25$:

$x^2 + 16x^2/9 = 25, \text{ so } 25x^2/9 = 25.$

$x = 3$ or $x = -3$, giving $y = 4$ or $y = -4$.

Final answer: $y = 4$ or $y = -4$

Question 84

A speed-time graph rises uniformly from 1.2 m/s to 4.8 m/s in 5 seconds, then falls uniformly to 0 in 2 seconds.
Find the total distance travelled.

Solution

The first part is a trapezium:
area $= \frac{1}{2}(1.2 + 4.8) \times 5 = 15$.
The second part is a triangle:
area $= \frac{1}{2} \times 2 \times 4.8 = 4.8$.
Total distance travelled $= 15 + 4.8 = 19.8$ m.
Final answer: 19.8 m

Question 85

Two positive numbers are in the ratio $2 : 3$.
Their squares differ by 45.
Find the numbers.

Solution

Let the numbers be $2k$ and $3k$.
Their squares differ by $9k^2 - 4k^2 = 5k^2$.
$5k^2 = 45,$ so $k^2 = 9$.
The numbers are positive, so $k = 3$.
The numbers are 6 and 9.
Final answer: 6 and 9

Question 86

Given that $2^m = 8^n$ and $m + n = 12$, work out m.

Solution

Since $8^n = 2^{3n}$, the equation $2^m = 8^n$ gives $m = 3n$.
Also $m + n = 12$.
$3n + n = 12,$ so $n = 3$.
Therefore $m = 9$.
Final answer: $m = 9$

Question 87

In triangle ABC, M and N are the midpoints of AB and AC.
If $\overrightarrow{BC} = 6a - 4b$, find $\overrightarrow{MN}$.

Solution

The line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
Therefore $MN = \frac{1}{2}BC$.
$MN = \frac{1}{2}(6a - 4b) = 3a - 2b$.
Final answer: $3a - 2b$