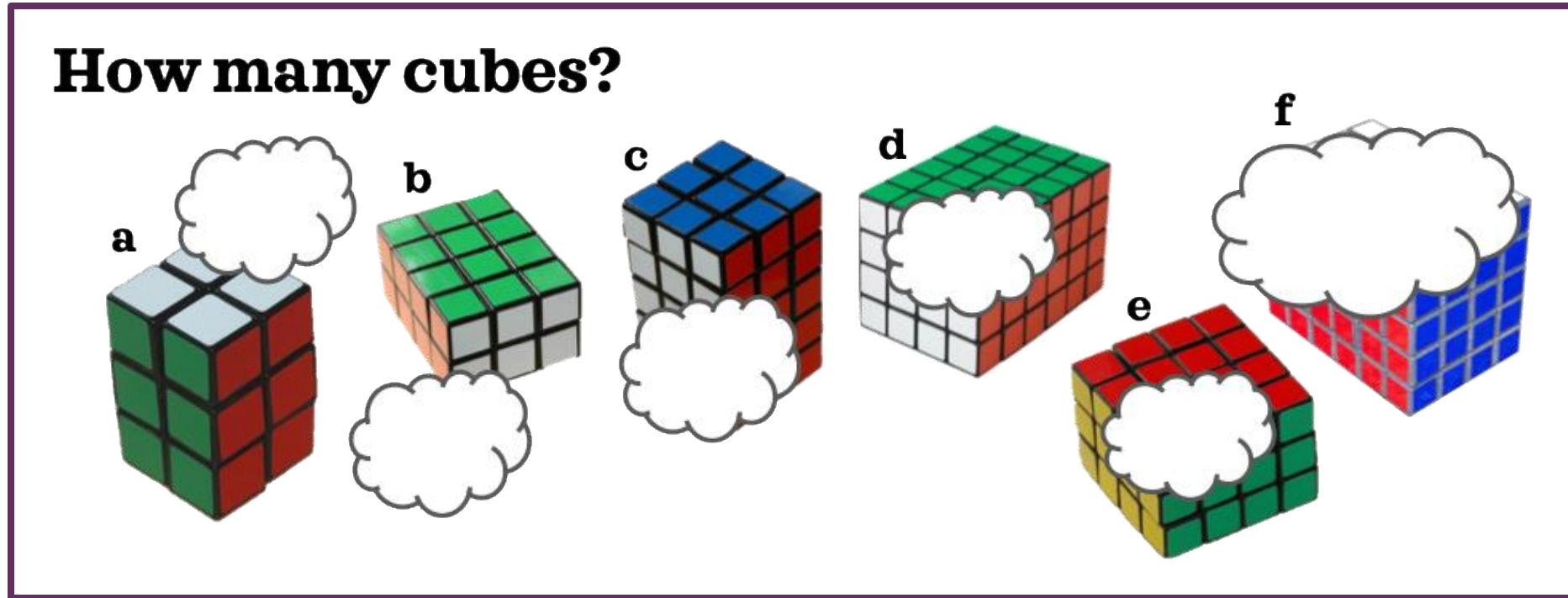


Please arrange yourself into a group of 3 or 4...



... and then discuss this task

Why might I have included it?

What thinking might it promote?

How might you use it?

How might you adapt it?

The Best That Has Been *Taught And Shared*

interwovenmaths.com/mc39

Nathan Day

[@nathanday.bsky.social](https://bsky.app/profile/nathanday.bsky.social)

interwovenmaths.com

In your group, discuss your task.
Think about these prompts.

Why might I have included it?

What principle(s) of task design does this task exhibit?

What makes this task stand out?

What thinking might it promote?

What does this task direct student's attention to?

What is the 'inner task'?

How might you use it?

What would you do to get the most out of this task?

What might come before/after using this task?

How might you adapt it?

How could you change this task to fit your class?

What might this task look like if it was about a different topic?

What principles did you think of? Which task(s) made you think of that principle?

Access point	Open vs Closed	Invitation to Fail	Increasing Constraints
Working Backwards	Multiple Answers / Approaches	Interweaving	Can do without reading
Opportunities to question preconceptions	Different representations	Requiring understanding	Multiple Strategies (Alg vs Num)
Thinking of the questions from the answers	Similarities and Differences	Variation	Misconceptions

What principles did you think of? Which task(s) made you think of that principle?

Intentionality	Floor-Ceiling Gap	Ladder Rungs?	Surprise
Elegance	Clarity	Purposefulness	Openness
Closedness	Discussability	Tricklessness	Tangibility
Simplicity	Journey	Self-checkingness	Prettiness

small data set average problems by Don Steward

A

A

averages and range (positive integers)

1) 3 numbers:
mean = 3
mode = 2

2) 3 numbers:
mean = 7
mode = 10

3) 3 numbers:
mean = 8
median = 10
range = 8

4) 3 numbers:
mean = 6
median = 7
range = 11

5) 3 numbers:
mode = 7
median = 7
mean = 6

6) 3 numbers:
mean = 13
range = 8

find three sets

7) 4 numbers:
mean = 4
mode = 1
median = 2

8) 4 numbers:
mean = 9
mode = 6
median = 7

9) 4 numbers:
mean = 6
median = $6\frac{1}{2}$
range = 11

10) 4 numbers:
mean = $7\frac{1}{2}$
mode = 6
median = 7

11) 4 numbers:
mean = 10
range = 12
mode = 13

12) 4 numbers:
mean = 8
range = 8
median = 7

find three sets

13) 4 numbers:
mean = 4
range = 6
median = 3

find two sets

14) 5 numbers:
range = 5
mean = 6
median = 7
mode = 8

15) 5 numbers:
range = 9
mean = 4
mode = 3

find two sets

16) 5 numbers:
range = 6
mean = 4
mode = 2

find two sets

17) 5 numbers:
range = 5
mean = 4
median = 3

find three sets

18) 5 numbers:
range = 10
mean = 7
mode = 7

find three sets

19) 5 numbers:
range = 5
mean = 5
mode = 5
median = 5

find two sets

20) 5 numbers:
range = 10
mean = 10
mode = 10
median = 10

find three sets

21) 5 numbers:
range = 10
mean = 4
mode = 1
median = 2

22) 6 numbers:
range = 10
mean = 4
mode = 1
median = 2

23) range = 10
mean = 4
median = 2
mode = 1

- a) 4 numbers
- b) 5 numbers
- c) 6 numbers


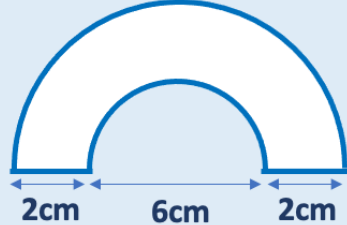
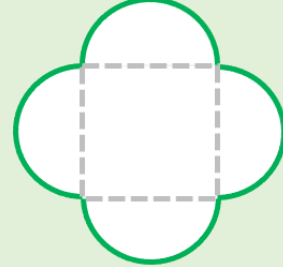
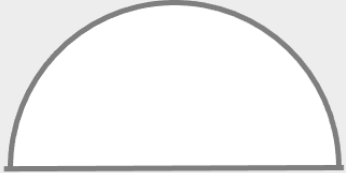
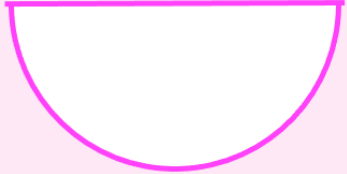

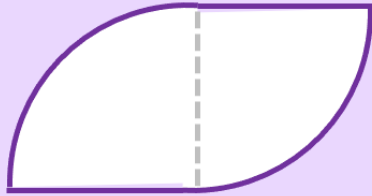
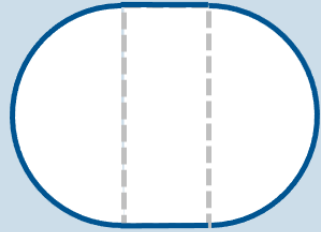
24) 5 numbers:
2, 5, n, 2n, 5n
mean =
 $2 \times \text{median} - 1$

find n

A

A




















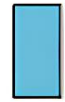


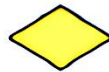














Which shapes have the same perimeter?

 <p>HEART: square with lengths of 8cm and 2 semi-circles</p>	 <p>ARCH: 2 concentric semi-circles</p>	 <p>FLOWER: square with lengths of 4cm and 4 semi-circles</p>	 <p>SEMI-CIRCLE: with diameter of 16cm</p>
 <p>BOWL: with diameter of 8cm</p>	 <p>CIRCLE: with diameter 8cm</p>	 <p>LEAF: Two quarter circles with radius of 4cm</p>	 <p>RACETRACK: 2 semi-circles with diameter of 8cm and a rectangle 8cm by 2cm</p>

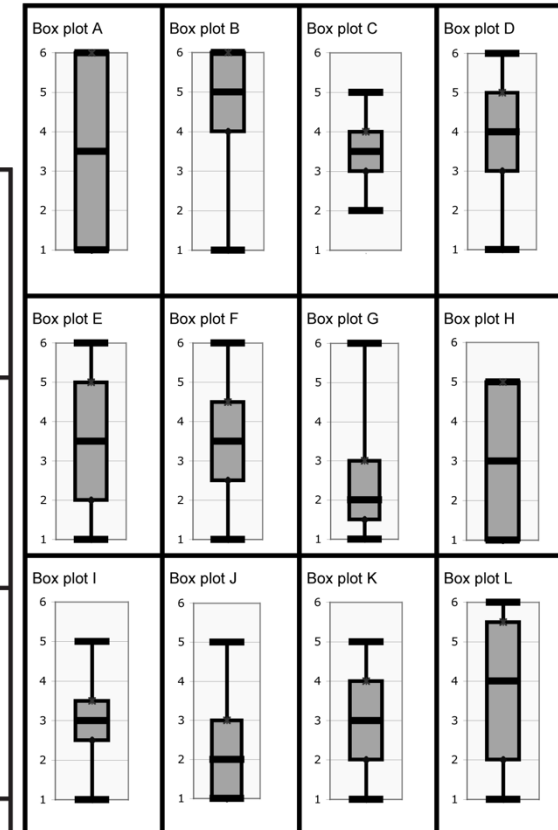
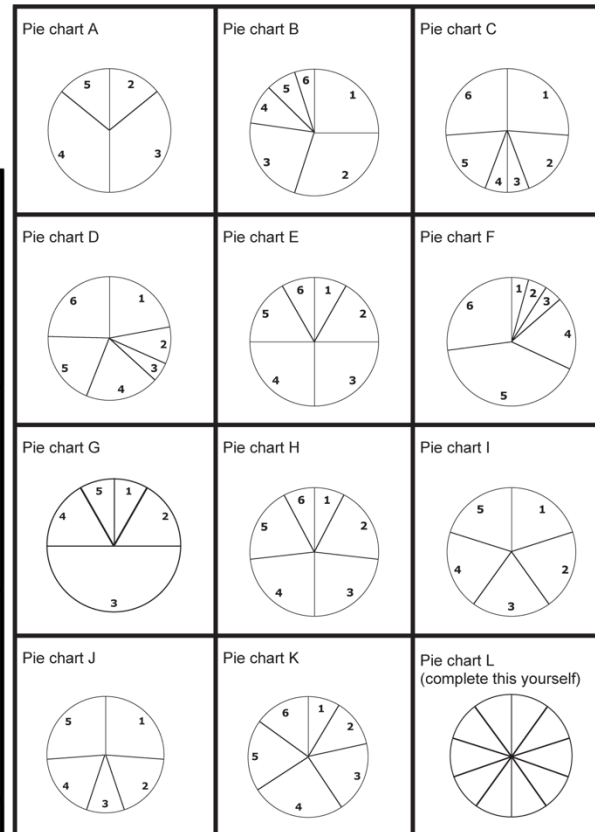
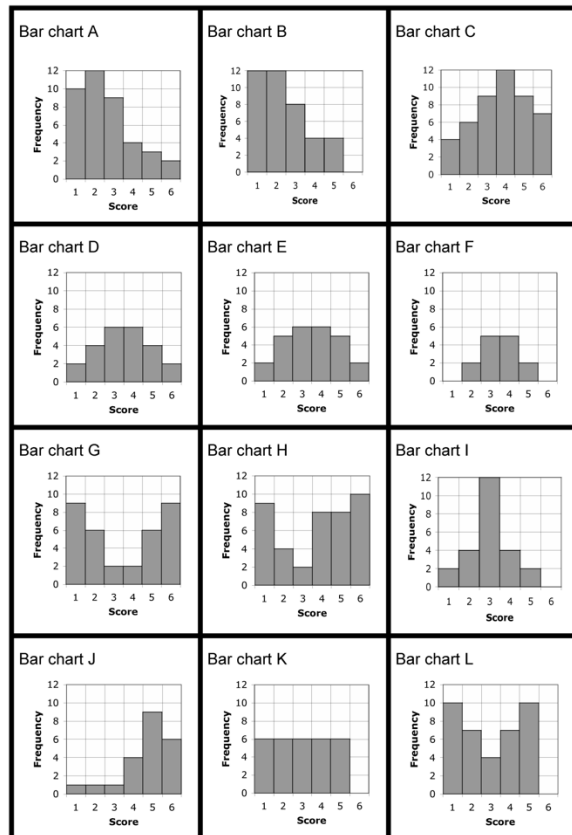
The coloured shapes stand for eleven of the numbers from 0 to 12.

Each shape is a different number.

Can you work out what they are from the multiplications below?

	x		x		=				x		=	
	x		=				x		=			
	x		=				x		=			
	x		=				x		=			
	x		=				x		=			
	x		=				x		=			

Match the card from each set.



Using the numbers 1 to 20, at most one time each, fill in the boxes to create equivalent expressions.

$$\left(2^{\square}\right)^{\square} = \frac{\left(2^{\square}\right)^{\square}}{\left(2^{\square}\right)^{\square}} = 2^{\square} \times 2^{\square} = \frac{2^{\square}}{2^{\square}}$$

Use these 12 numbers, once each, in the gaps below.

10, 20, 25, 35, 40, 50, 60, 70, 75, 80, 90, 100

£ _____ increased by _____ % = £ _____

£ _____ increased by _____ % = £ _____

£ _____ decreased by _____ % = £ _____

£ _____ decreased by _____ % = £ _____

Checkpoint 13: Equivalent fractions

- a) Complete the missing boxes to find fractions equivalent to $\frac{3}{21}$.

$$\frac{3}{21} = \frac{5}{\square} \qquad \frac{3}{21} = \frac{6}{\square}$$

- b) Which did you find easier to do? Why?
- c) Find three more fractions equivalent to $\frac{3}{21}$. What is your strategy?



$3 \times 97 = 291$. Use this fact to find these equivalent fractions:

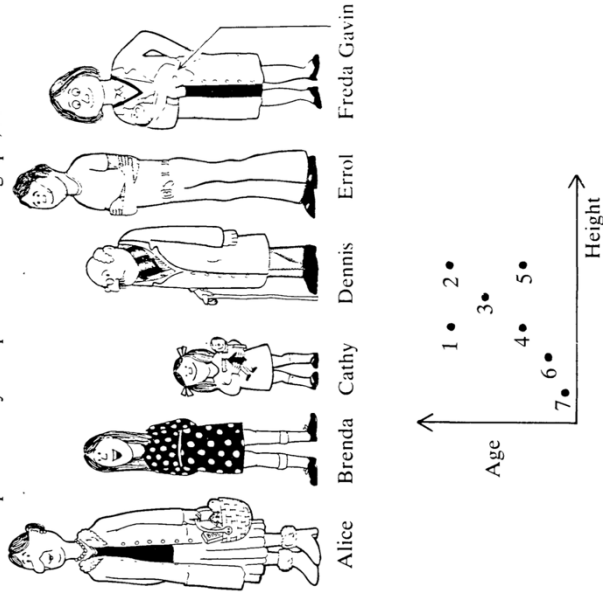
$$\frac{3}{291} = \frac{10}{\square} \qquad \frac{3}{4} = \frac{291}{\square}$$

A1 INTERPRETING POINTS

As you work through this booklet, discuss your answers with your neighbours and try to come to some agreement.

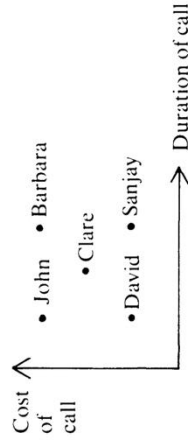
1. The Bus Stop Queue

Who is represented by each point on the scattergraph, below?



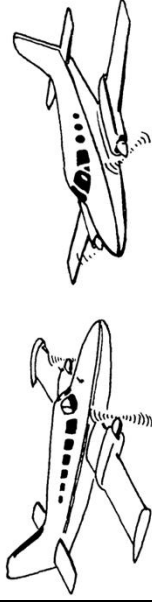
3. Telephone Calls

One weekend, five people made telephone calls to various parts of the country. They recorded both the cost of their calls, and the length of time they were on the telephone, on the graph below:

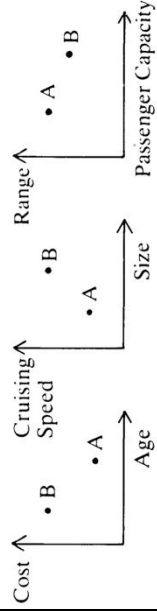


- * Who was ringing long-distance? Explain your reasoning carefully.
- * Who was making a local call? Again, explain.
- * Which people were dialling roughly the same distance? Explain.
- * Copy the graph and mark other points which show people making local calls of different durations.
- * If you made a similar graph showing every phone call made in Britain during one particular week-end, what would it look like? Draw a sketch, and clearly state any assumptions you make.

2. Two Aircraft



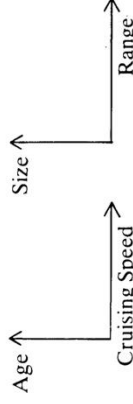
The following quick sketch graphs describe two light aircraft, A and B: (note: the graphs have *not* been drawn accurately)



The first graph shows that aircraft B is more expensive than aircraft A. What else does it say?

- * Are the following statements true or false?
 - The older aircraft is cheaper?*
 - The faster aircraft is smaller?*
 - The larger aircraft is older?*
 - The cheaper aircraft carries fewer passengers?*

* Copy the graphs below. On each graph, mark and label two points to represent A and B.

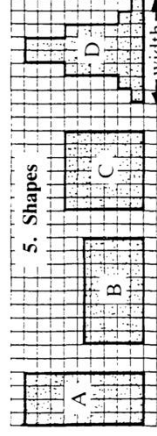
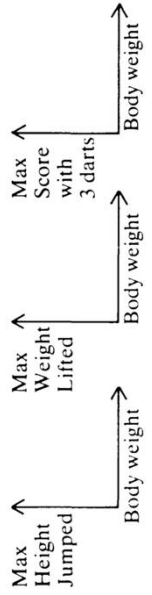


4. Sport

Suppose you were to choose, at random, 100 people and measure how heavy they are. You then ask them to perform in 3 sports;

High Jumping, Weight Lifting and Darts.

Sketch scattergraphs to show how you would expect the results to appear, and explain each graph, underneath. Clearly state any assumptions you make . . .



These four shapes each have an area of 36 square units.

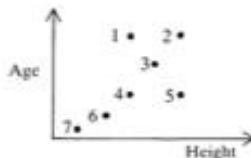
- * Label four points on the graph below, with the letters A, B, C and D.
- * Can you draw a fifth shape, with an area of 36 square units, to correspond to the other point? Explain.
- * Draw a scattergraph to show every *rectangle* with an area of 36 square units.
- * Finally, what happens if you include *all* shapes, with the same area, on your graph?

A1 INTERPRETING POINTS

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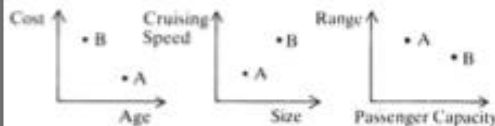


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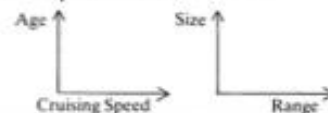


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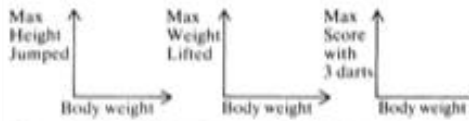


4. Sport

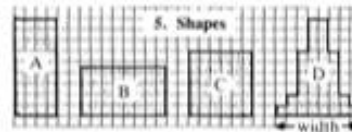
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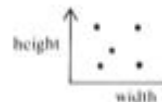


5. Shapes

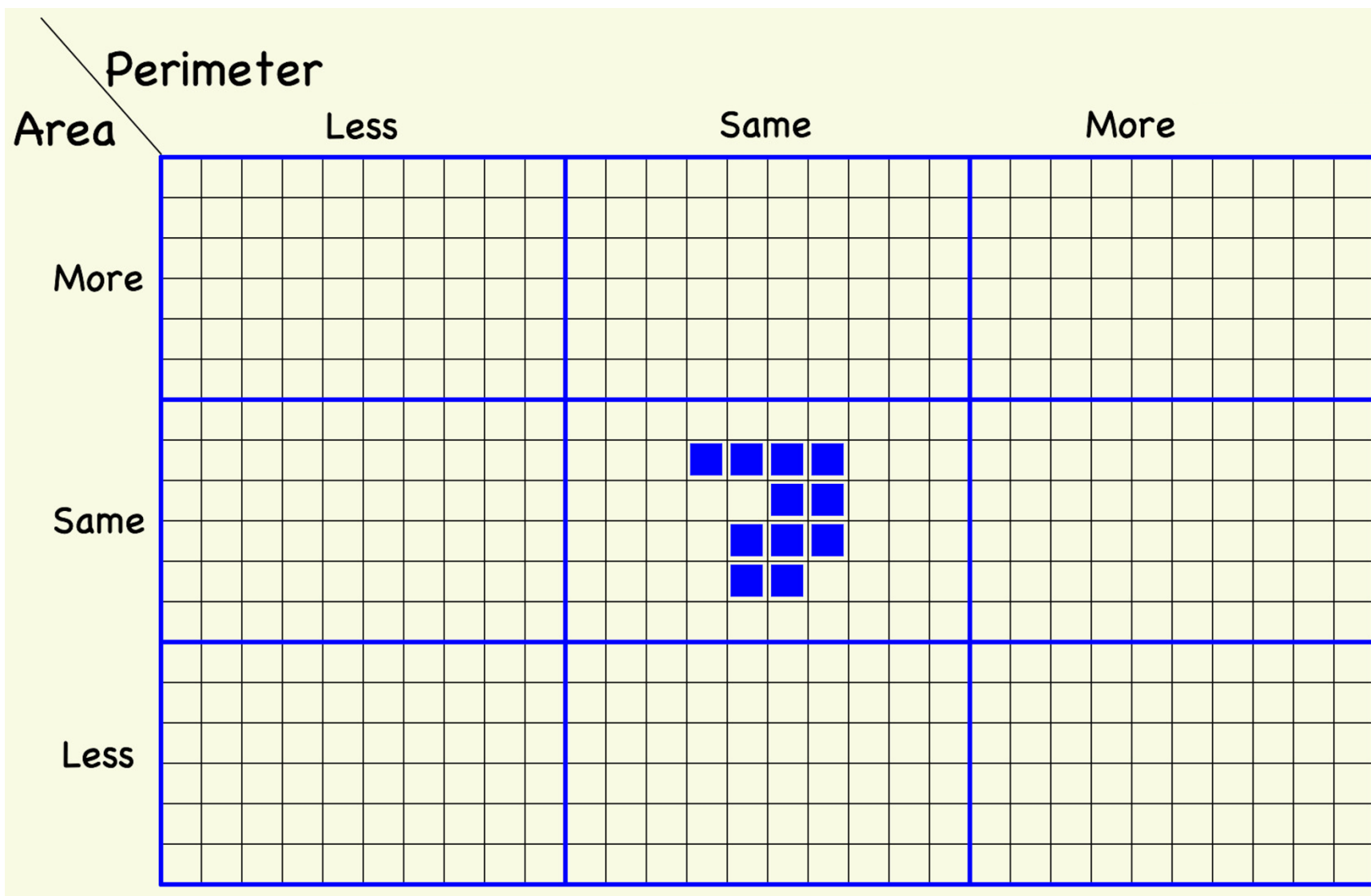


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- Can you draw a fifth shape, with an area of 36 square units, to correspond to the other point? Explain.
- Draw a scattergraph to show every rectangle with an area of 36 square units.
- Finally, what happens if you include all shapes, with the same area, on your graph?



More or Less Perimeter and Area Grid by John Mason



Which is bigger, $3n$ or $n + 3$?

$2^1 = 2$	$2^{11} = 2,048$	$2^{21} = 2,097,152$
$2^2 = 4$	$2^{12} = 4,096$	$2^{22} = 4,194,304$
$2^3 = 8$	$2^{13} = 8,192$	$2^{23} = 8,388,608$
$2^4 = 16$	$2^{14} = 16,384$	$2^{24} = 16,777,216$
$2^5 = 32$	$2^{15} = 32,768$	$2^{25} = 33,554,432$
$2^6 = 64$	$2^{16} = 65,536$	$2^{26} = 67,108,864$
$2^7 = 128$	$2^{17} = 131,072$	$2^{27} = 134,217,728$
$2^8 = 256$	$2^{18} = 262,144$	$2^{28} = 268,435,456$
$2^9 = 512$	$2^{19} = 524,288$	$2^{29} = 536,870,912$
$2^{10} = 1024$	$2^{20} = 1,048,576$	$2^{30} = 1,073,741,824$

Without using a calculator or paper, can you mentally compute these expressions?

- 32×16
- $4 \times 64 \times 1024$
- $4 \times 16 \times 32 \times 64$
- $1024 \div 64$
- $1\,048\,576 \div 32\,768$
- $1\,073\,741\,824 \div 64$
- 128^2
- 1024^3
- $\sqrt{65\,536}$
- $\sqrt[3]{134\,217\,728}$
- $\sqrt[5]{1\,073\,741\,824}$
- $(1\,048\,576)^{\frac{1}{2}}$
- $(262\,144)^{\frac{2}{3}}$
- $\frac{524\,288}{(16\,384)^{\frac{1}{2}}}$
- $\frac{(67\,108\,864)^{\frac{1}{2}}}{16^3(256)^{\frac{1}{4}}}$
- $\frac{1}{4}(65\,536)$

9 Four Fours

Using exactly four **4**s and no other digits, we can make **0**:

$$0 = 44 - 44$$

$$0 = (4 - 4) (4! + .4)$$

$$0 = 4^{\sqrt{4}} - \sqrt{4}^4$$

$$0 = \log_4 4 - \frac{4}{4}$$

Using exactly four **4**s, can you make **1? 2? 3? 4? 5? ...?**

Source: Martin Gardner

[Play With Your Math.com](https://playwithyourmath.com)

This problem dates back to at least 1881. Details of the history can be found here:
<https://pballew.blogspot.com/2018/12/before-there-were-four-fours-there-were.html>

Which equation matches which graph?

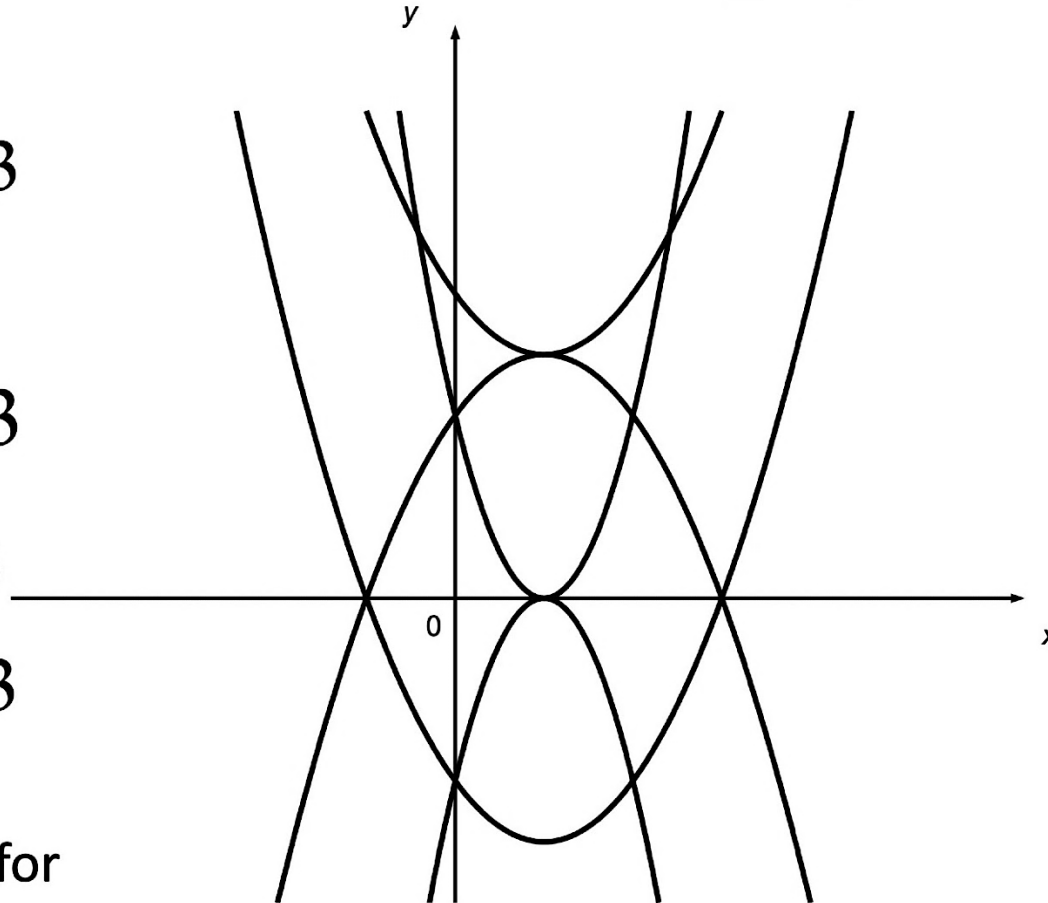
$$y = 6x - 3x^2 - 3$$

$$y = x^2 - 2x + 5$$

$$y = x^2 - 2x - 3$$

$$y = 2x - x^2 + 3$$

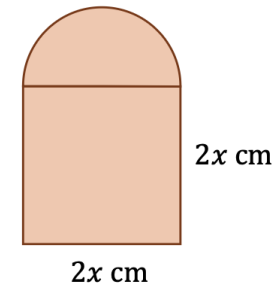
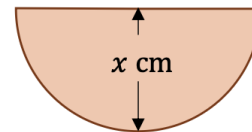
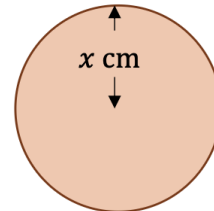
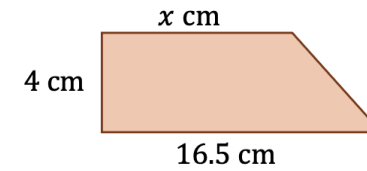
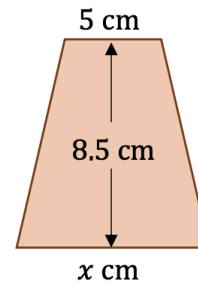
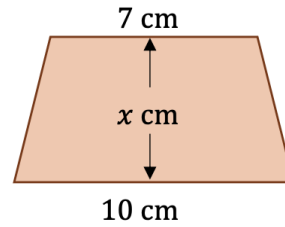
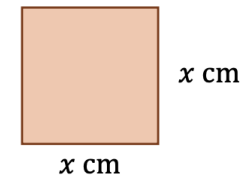
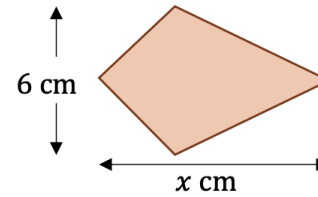
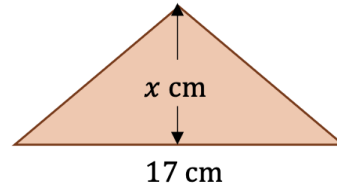
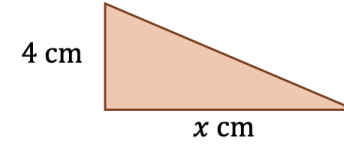
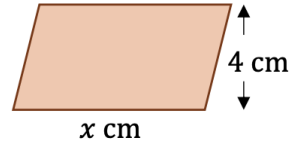
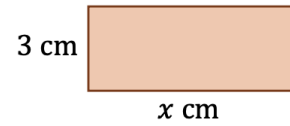
$$y = 3x^2 - 6x + 3$$



Explain the reasons for
your choices

Each of these 2D shapes has an area of 51 cm^2 .

Find the value of the missing length x in each case.



$$\text{Circumference: } C = \pi d$$

$$\text{Area: } A = \pi r^2$$

1. Each of these calculations would find the perimeter or area of a shape involving part of a circle.

Sketch a fully-labelled shape that matches each calculation

a) $A = \pi \times 2^2$

b) $A = \frac{1}{2} \times \pi \times 3^2$

c) $P = \frac{1}{2} \times \pi \times 6 + 6$

d) $A = \frac{1}{2} \times \pi \times 3^2 + 8 \times 6$

e) $P = \frac{1}{2} \times \pi \times 6 + 5 + 5$

f) $P = \frac{1}{2} \times \pi \times 6 + 8 + 6 + 8$

g) $P = \frac{1}{4} \times \pi \times 6 + 7 + 5$

h) $A = \frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times 6 \times 4$

i) $A = 4 \times 8 - \frac{1}{4} \times \pi \times 4^2$

j) $P = \frac{1}{4} \times \pi \times 8 + 8 + 4 + 4$

2. Draw as many possible shapes that you can with

a) *Perimeter* of exactly $2\pi + 8$

b) *Area* of exactly $2\pi + 8$

Can you find any shapes that no-one else has thought of?

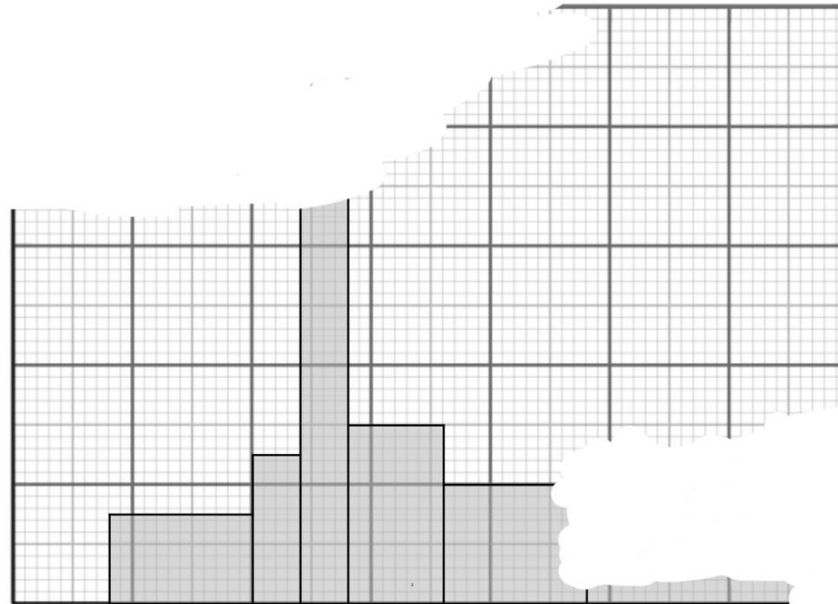
Can you find any shapes that solve both?

Erin has collected the following data about 63 members of her local running club and printed off the frequency table and histogram.

The printer is running out of ink – so it hasn't printed clearly

Complete the table and the histogram for Erin.

PB for 10 km (minutes)	Frequer
$34 \leq t < 40$	
	15
	12
$48 \leq t < 54$	
$54 \leq t \leq 64$	



1) Write a *single-line* calculation to represent each situation

(a) Hazma bought 4 shirts for £15 each and 2 pairs of trousers for £25 each. How much did he spend?

(b) Mark earns £100 a day, working 5 days a week. On two of the days he buys lunch, costing £4 each time. How much does he have left over?

(c) A shirt normally costs £80. Today there a £35 discount. A man and his dad buy 4 shirts and then share the cost. How much does each pay?

(d) There are three square-shaped rooms in my house. Two of them have 5m sides and the other 4m sides. What's their total floor area?

(e) Louise pays £20 for materials to make earrings. She makes 10 earrings and sells 7 for £5 and 3 for £2. How much profit does she make?

2) Write story problems which could match up with each calculation

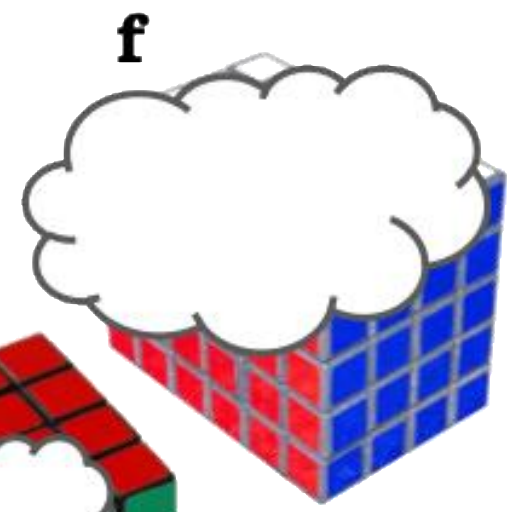
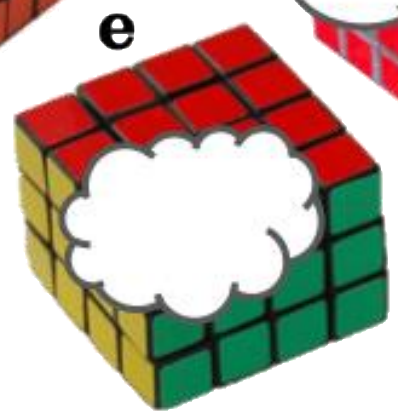
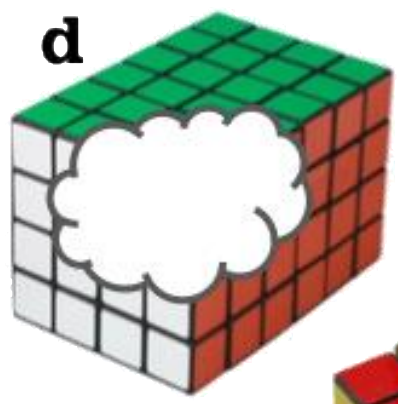
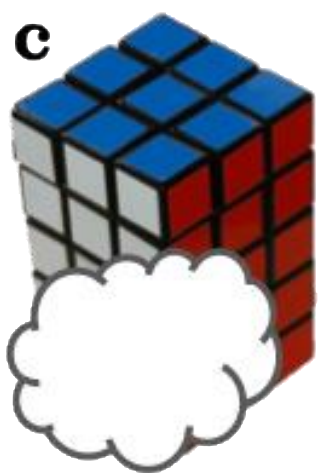
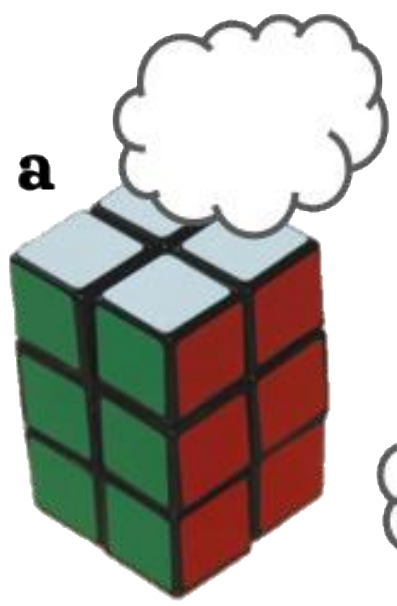
(a) $3 \times 2 + 4 \times 5$

(b) $12 - 5 \times 2$

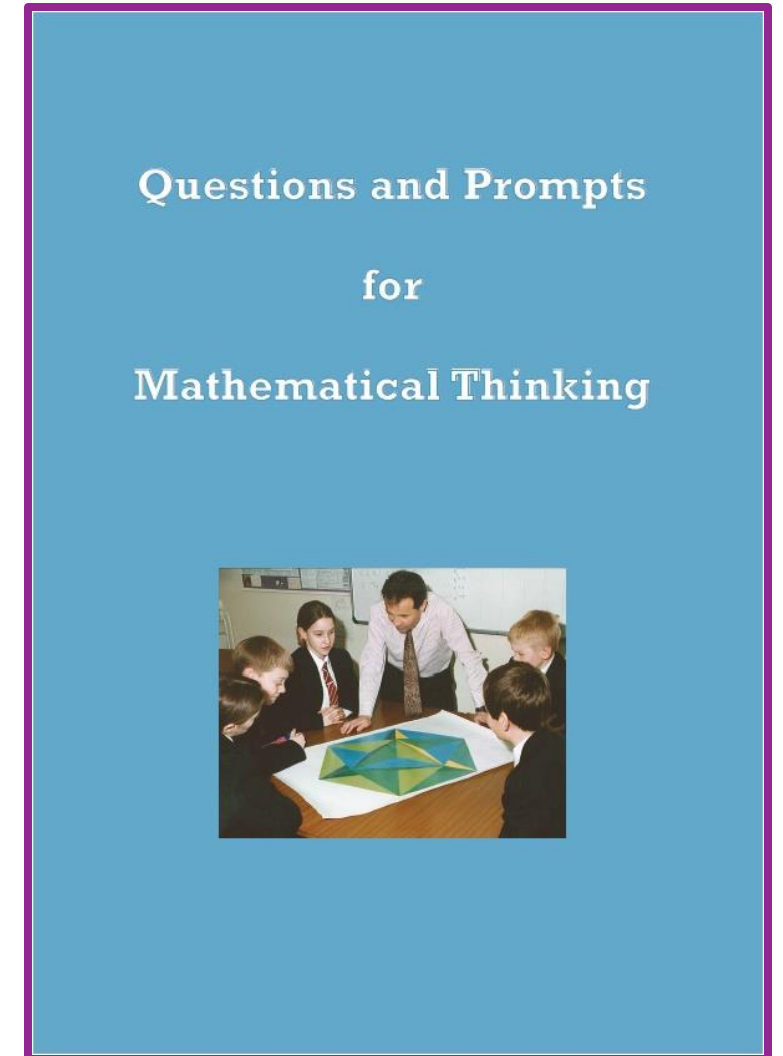
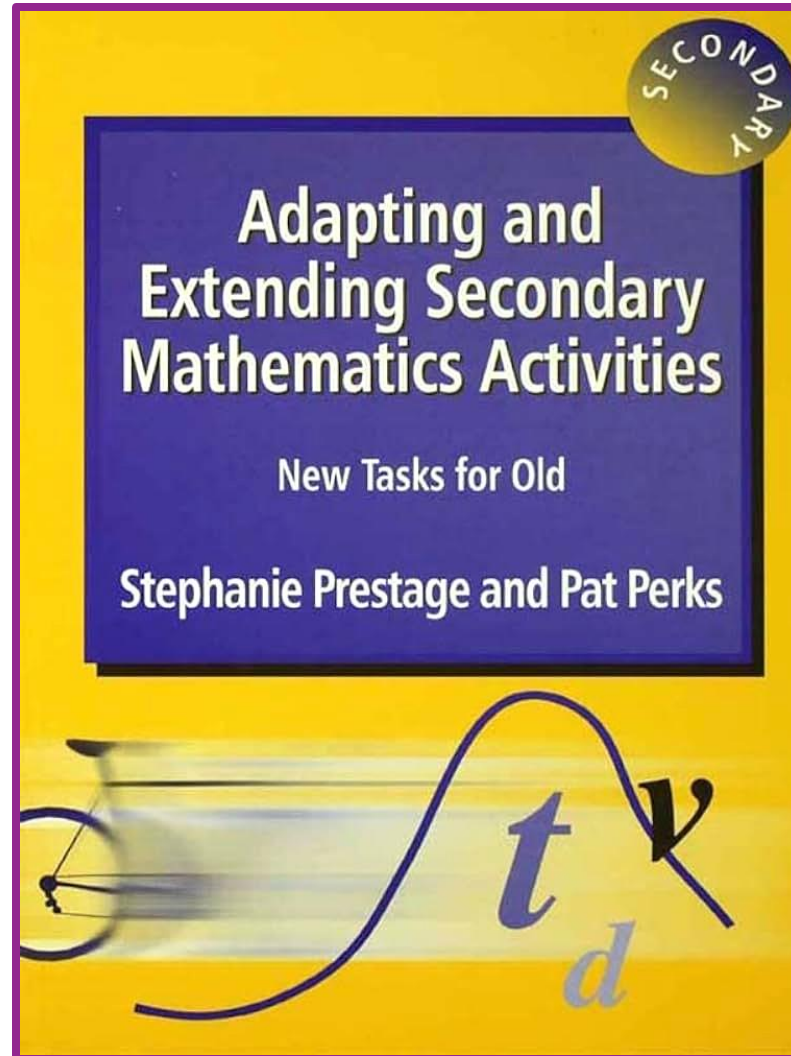
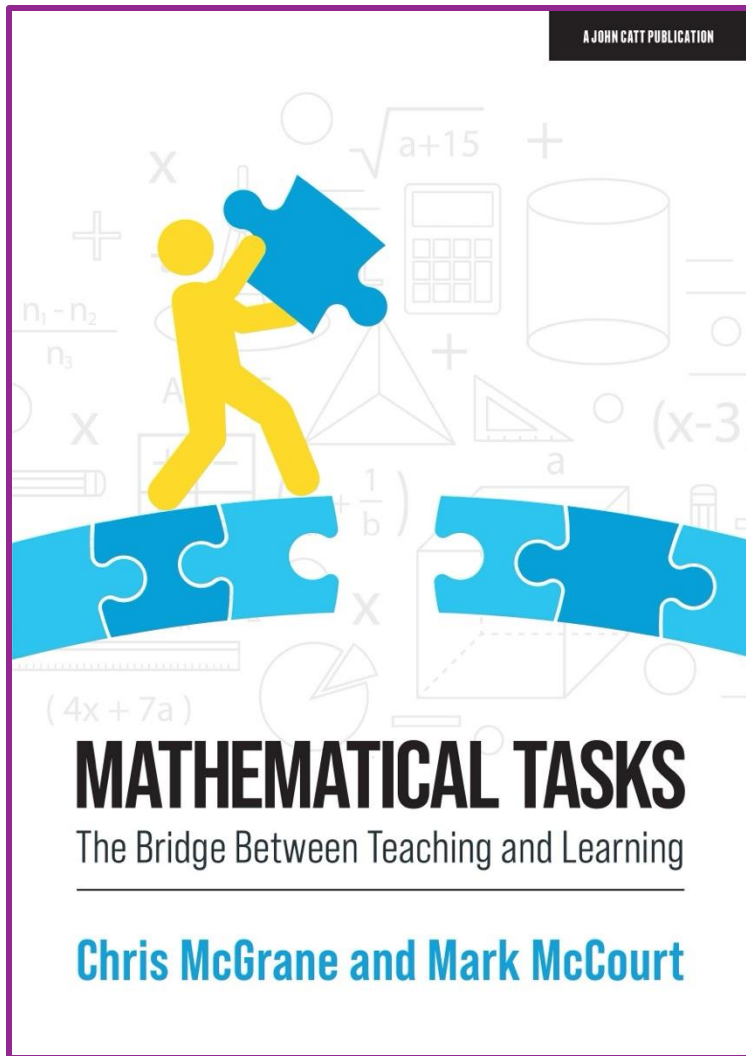
(c) $3 \times 2(4 + 7)$

(d) $5^2 - 4^2$

How many cubes?



Further Reading



Questions to leave with...

How can you adapt your existing resources to include these ideas more?

How can you adapt your teaching to include these ideas more, without changing your resources?