Introducing Sequences And Series

an interwoven approach

Do Now

Calculate:

$$\int_0^{\frac{1}{10}} 1 + x + x^2 + x^3 \, dx$$

Solve:

$$e^{2x} - e^x = \frac{10}{81}$$

Do Now

Calculate:

$$\int_0^{\frac{1}{10}} 1 + x + x^2 + x^3 \, dx$$

$$= \left[x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 \right]_0^{\frac{1}{10}}$$

$$= \frac{1}{10} + \frac{1}{200} + \frac{1}{3000} + \frac{1}{40000}$$

$$= 0.105358 \text{ (6 d.p)}$$

Solve:

$$e^{2x} - e^x = \frac{10}{81}$$

$$81e^{2x} - 81e^x - 10 = 0$$

$$(9e^x + 1)(9e^x - 10) = 0$$

$e^x = \frac{10}{9}$ (reject $e^x = -\frac{1}{9}$ as $e^x > 0$)

$x = \ln(\frac{10}{9}) = 0.105361$ (6 d.p)

Task 1

Calculate:

a. $\int_0^{\frac{1}{10}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

b. $\int_0^{\frac{1}{4}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

c. $\int_0^{\frac{1}{2}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

d. $\int_0^{\frac{1}{n}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

Expand and simplify:

a. $(1 - x)(1 + x + x^2 + x^3 + \cdots)$

b. $(1 + x + x^2 + \cdots)(1 + x + x^2 + \cdots)$

Task 1

Calculate:

a. $\int_0^{\frac{1}{10}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

$0.1111 \ldots = \frac{1}{9}$

b. $\int_0^{\frac{1}{4}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

$0.3333 \ldots = \frac{1}{3}$

c. $\int_0^{\frac{1}{2}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

1

d. $\int_0^{\frac{1}{n}} 1 + 2x + 3x^2 + 4x^3 + \cdots dx$

$\frac{1}{n-1}$

Expand and simplify:

a. $(1 - x)(1 + x + x^2 + x^3 + \cdots)$

1

b. $(1 + x + x^2 + \cdots)(1 + x + x^2 + \cdots)$

$1 + 2x + 3x^2 + \cdots$

Task 2

Achilles and a tortoise are in a race.
Both start from rest.

Achilles runs at 10 metres per second.

The tortoise walks at 10 metres per minute.

However, the tortoise started the race 600 m ahead of Achilles.

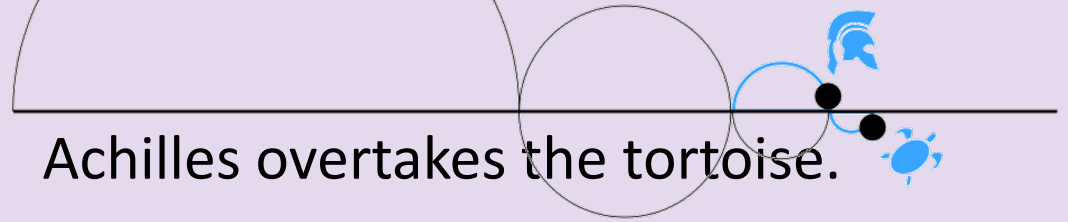
The tortoise gloats to Achilles:
'You can never overtake me, since every time you reach where I was, I'll have moved a bit further.'

Find the time when:

a. Achilles reaches the tortoise's starting point.

b. Achilles reaches where the tortoise was when Achilles reached the tortoise's starting point.

c. Achilles reaches where the tortoise was when Achilles reached the point where the tortoise was when Achilles reached the tortoise's starting point.



d. Achilles overtakes the tortoise.

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Find the time when:

a. Achilles reaches the tortoise's starting point.

60 seconds

b. Achilles reaches where the tortoise was when Achilles reached the tortoise's starting point.

61 seconds

c. Achilles reaches where the tortoise was when Achilles reached the point where the tortoise was when Achilles reached the tortoise's starting point.

$61\frac{1}{60}$ seconds

d. Achilles overtakes the tortoise.

$61\frac{1}{59}$ seconds

Task 2

Achilles and a tortoise are in a race.
Both start from rest.

Achilles **accelerates** at 10 ms^{-2}.

The tortoise **accelerates** at 1 ms^{-2}.

However, the tortoise started the race 600 m ahead of Achilles.

The tortoise gloats to Achilles:
'You can never overtake me, since every time you reach where I was, I'll have moved a bit further.'

Find the time when:

a. Achilles reaches the tortoise's starting point.

b. Achilles reaches where the tortoise was when Achilles reached the tortoise's starting point.

c. Achilles reaches where the tortoise was when Achilles reached the point where the tortoise was when Achilles reached the tortoise's starting point.

d. Achilles overtakes the tortoise.

Task 2

Achilles and a tortoise are in a race.
Both start from rest.

Achilles **accelerates** at 10 ms^{-2}.

The tortoise **accelerates** at 1 ms^{-2}.

However, the tortoise started the race 600 m ahead of Achilles.

The tortoise gloats to Achilles:
'You can never overtake me, since every time you reach where I was, I'll have moved a bit further.'

Find the time when:

a. Achilles reaches the tortoise's starting point.

10.95 seconds

b. Achilles reaches where the tortoise was when Achilles reached the tortoise's starting point.

11.49 seconds

c. Achilles reaches where the tortoise was when Achilles reached the point where the tortoise was when Achilles reached the tortoise's starting point.

11.54 seconds

d. Achilles overtakes the tortoise.

11.55 seconds

Task 3

Anne and Bob are playing a game.

They have a biased coin that lands on heads with a probability of $\frac{2}{3}$.

They alternate flipping the coin, with Anne going first.

A player gets a point if the coin lands on heads on their turn.

Find the probability that:

a. Anne wins a point on exactly 7 of her first 10 turns?

b. Anne wins the first point of the game on her second turn?

c. Anne wins the first point of the game?

Task 3

Anne and Bob are playing a game.

They have a biased coin that lands on heads with a probability of $\frac{2}{3}$.

They alternate flipping the coin, with Anne going first.

A player gets a point if the coin lands on heads on their turn.

Find the probability that:

a. Anne wins a point on exactly 7 of her first 10 turns?

0.26012

b. Anne wins the first point of the game on her second turn?

$\frac{2}{27}$

c. Anne wins the first point of the game?

$\frac{3}{4}$

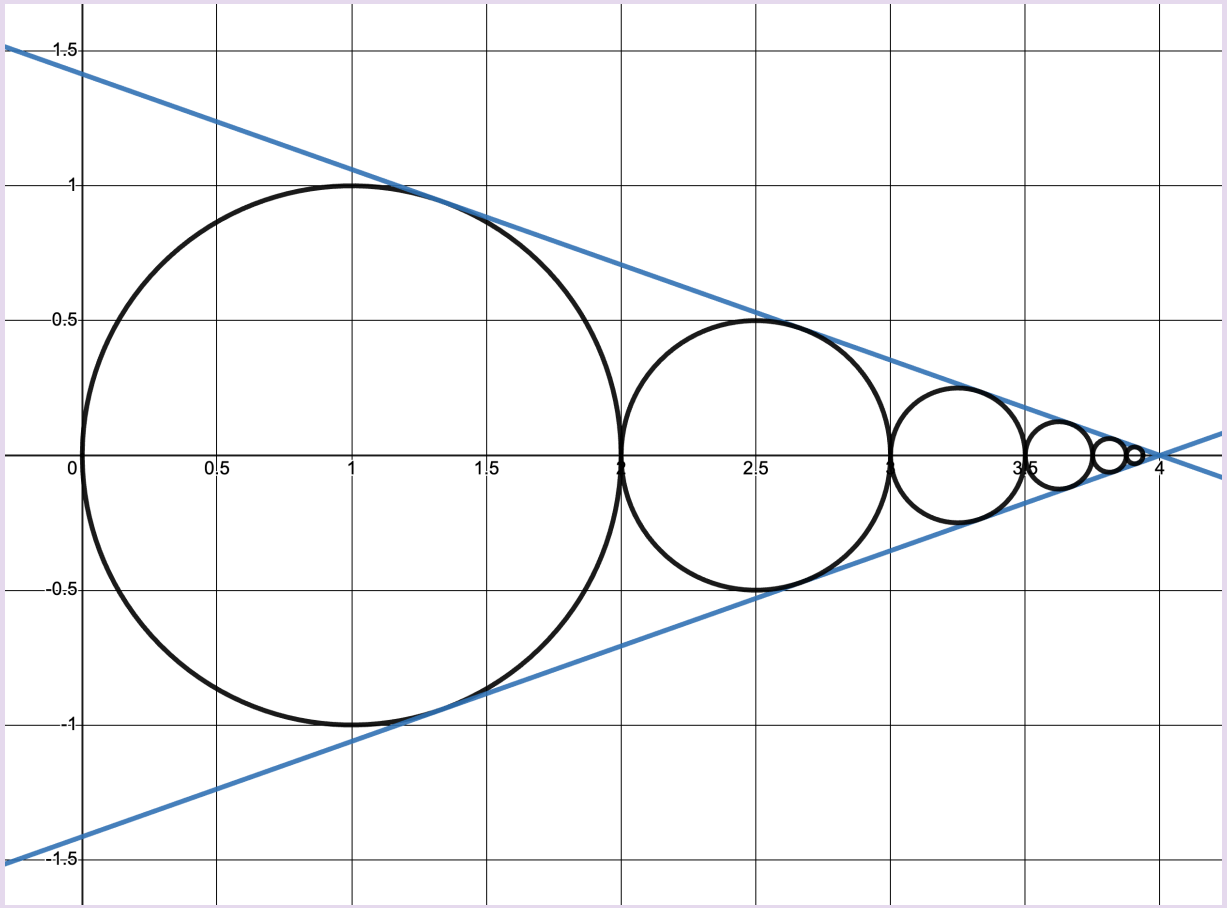
Task 3

Anne claims that the coin does not, in fact, have a probability of $\frac{2}{3}$ of landing on heads.

When she flips the coin 25 times, it lands on heads 21 times.

Test Anne's claim with a 10% significance level.

Task 4



a. Find the length of the radius of each circle.

$2, 1, 0.5, 0.25, \ldots$

b. Find the equation of each circle.

$(x-1)^2+y^2=1$

$\left(x-\frac{5}{2}\right)^2+y^2=\frac{1}{4}$

$\left(x-\frac{13}{4}\right)^2+y^2=\frac{1}{16}$

$\left(x-\frac{29}{8}\right)^2+y^2=\frac{1}{64}$

$\left(x-\frac{61}{16}\right)^2+y^2=\frac{1}{256}$

c. Find the equations of the blue lines.

$y=\pm\sqrt{8}(x-4)$