

# Challenge Expected.

insert tagline here

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**Nathan Day**

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# A task to get us started...

Fill in the gaps using either  or  :

$$8(x \square 3) \square 6(x \square 7) = 2(x \square 9)$$

**a** Can you make an equation with...

i) ... a positive solution

ii) ... a negative solution

iii) ... the solution  $x = 0$

iv) ... no solutions

v) ... infinitely many solutions

**b** How many different solutions can you make in total?

# What makes a maths question challenging?

# Challenge can be...

<b>Mechanical</b> How complicated the numbers are.	<b>Procedural</b> How many steps are needed.	<b>Conceptual</b> How difficult the topic is.	<b>Textual</b> How hard it is to understand the question's wording.
<b>Contextual</b> How much real-world knowledge is needed.	<b>Abstractual</b> How abstract the question is.	<b>Structural</b> How structured is the problem.	<b>Familial</b> How familiar does the question feel.
<b>Tricky</b> Is spotting a trick required.	<b>Intuitial</b> How intuitive is the solution.	<b>Memorial</b> How hard is it to recall necessary knowledge.	<b>Exactitudinal</b> How precise do you have to be.

**What makes a maths question challenging?**

If it requires a deeper understanding of the topic.

**How can we make questions  
that are more challenging?**

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**How can we make challenging  
questions more accessible?**

# How can we make questions that are more challenging?

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Require justifications





**Sam Blatherwick**

@blatherwick\_sam



my new year 10s hate showing their working out

so I have taken away their beloved answers

1. **Explain** why the lowest common multiple of 75 and 60 is 300
2. **Explain** why the lowest common multiple of 450 and 288 is 7200
3. **Explain** why the highest common factor of 2400 and 2850 is 150
4. Denise says she has a quick way to work out a lowest common multiple. For example: 12 and 18  
“Write it as a fraction and cancel it down  $\frac{12}{18} = \frac{2}{3}$   
I needed to divide them both by 6, so I do  $2 \times 3 \times 6 = 36$  which is the lowest common multiple”  
Try this method with **15 and 21** and with **75 and 60**. Why does it work? Can you **explain**?
5. The lowest common multiple of 45 and another number is 540. Work out what the other number could be. Are there any other possibilities?



<p>Show that...</p> <p><b>A</b> is 80% of <b>C</b></p>	<p>Show that...</p> <p><b>A + B</b> is a multiple of 7</p>	<p>Show that...</p> <p><b>A</b> is both the mean and the median of the three numbers</p>
<p>Show that...</p> <p><b>B</b> has more factors than <b>C</b> if and only if <math>r &gt; q</math></p>	<p><b>A</b> = <math>2^{p+2} \times 3^q \times 5^r</math></p> <p><b>B</b> = <math>2^p \times 3^{q+1} \times 5^r</math></p> <p><b>C</b> = <math>2^p \times 3^q \times 5^{r+1}</math></p>	<p>Show that...</p> <p>The gradient of the line going through <b>(A, C)</b> and <b>(B, B)</b> is 2</p>
<p>Show that...</p> <p><b>A, B</b> and <b>C</b> can be the first three terms of an arithmetic sequence</p>	<p>Show that...</p> <p><b>A, B</b> and <b>C</b> can be sides lengths of a right-angled triangle</p>	<p>Show that...</p> <p><math>3(\mathbf{C} - \mathbf{A}) = \mathbf{B}</math></p>

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Require justifications

Generalise

### Ratio

$\widehat{ADB}$  and  $\widehat{AEB}$  are in the ratio 2 : 7.

What is  $\widehat{ADB}$ ?

### Equations

$\widehat{ABC}$  is  $10^\circ$  greater than  $\widehat{ADB}$ .

What is  $\widehat{ABC}$ ?

### Percentages

$\widehat{ABC}$  is 25% of the size of  $\widehat{ACB}$ .

What is  $\widehat{ABC}$ ?

## Circle Theorems with...

$C$  is the centre of the circle, all other points are on the circumference.

### Bounds

$\widehat{ACB}$  is  $30^\circ$ , to the nearest  $10^\circ$ .

What is the error interval for  $\widehat{ADB}$ ?

### Averages

The mean of  $\widehat{ACB}$  and  $\widehat{ADB}$  is  $24^\circ$ .

What is  $\widehat{ADB}$ ?

### Sequences

The angles of ABDE form an arithmetic sequence.

The smallest angle is  $45^\circ$ .  
What is the second smallest angle?

Rory ran to school at 6 mph, and then ran home the same way at 4 mph. What was Rory's average speed?



What is the density of an alloy made from equal masses of gallium and barium?

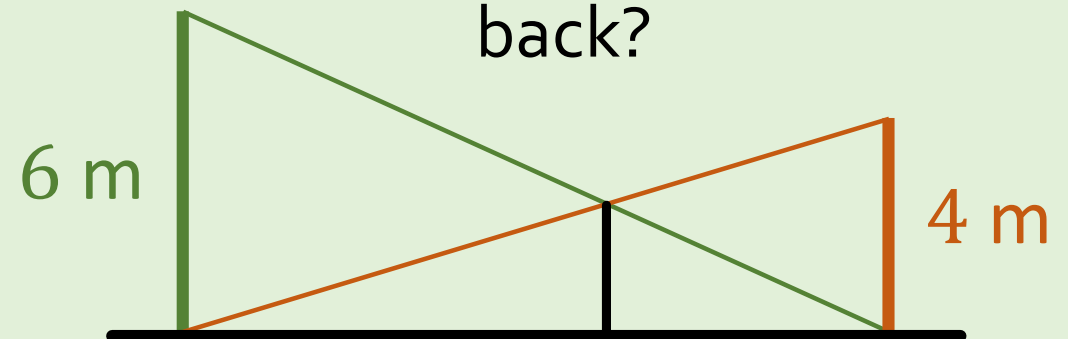
Metal	Density
Gallium	6 g/cm <sup>3</sup>
Barium	4 g/cm <sup>3</sup>

## Why do these questions all have the same answer?

Anne could paint a room in 6 hours. Bob could paint the room in 4 hours. How long would they take to paint two coats of the room, working together?



An engineer has to fix a fault where these two wires cross. How far does she have to climb to get to the fault and back?



# How can we make questions that are more challenging?

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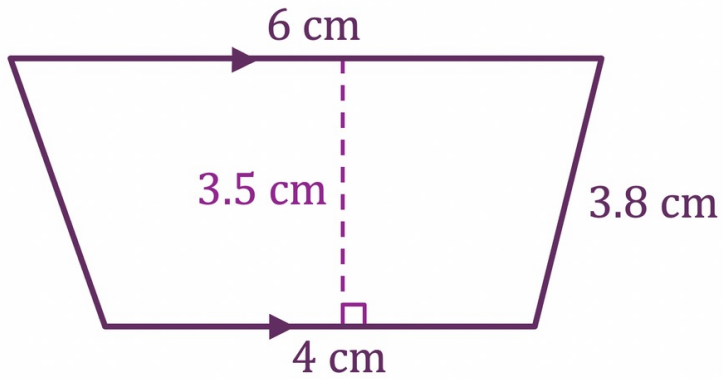
Require justifications

Generalise

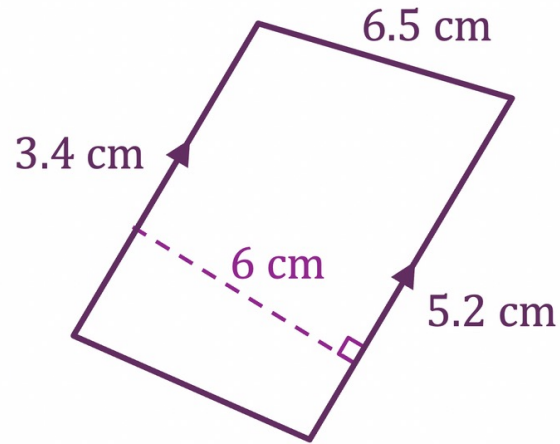
Make things messy

Find the area of each trapezium.

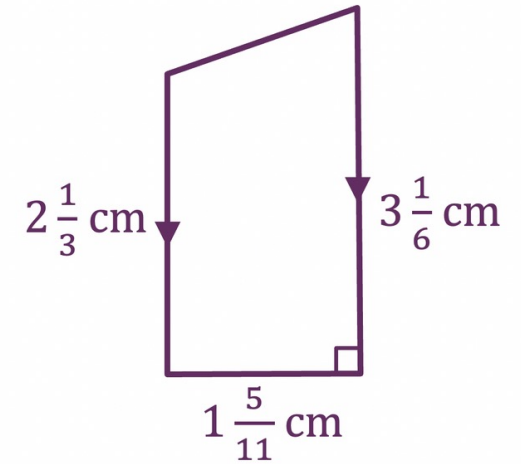
a)



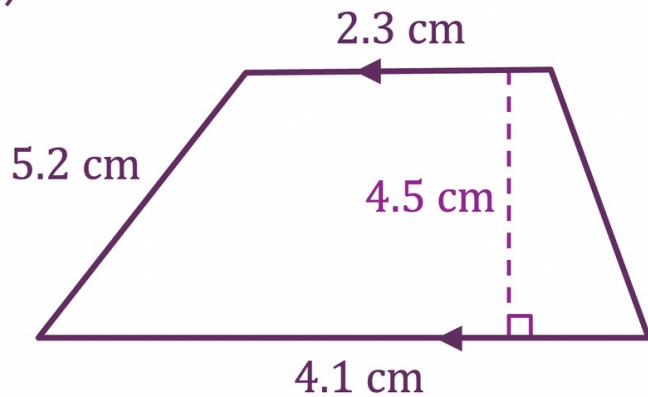
b)



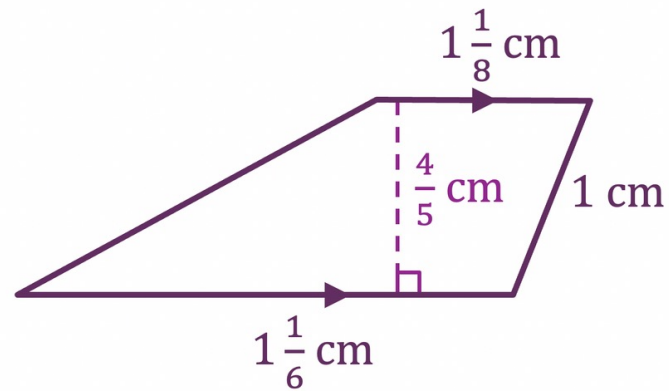
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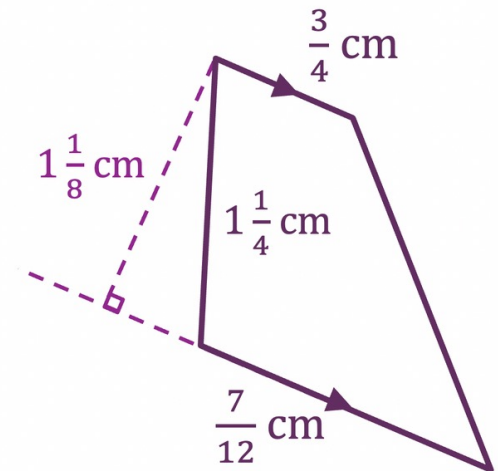
d)



e)



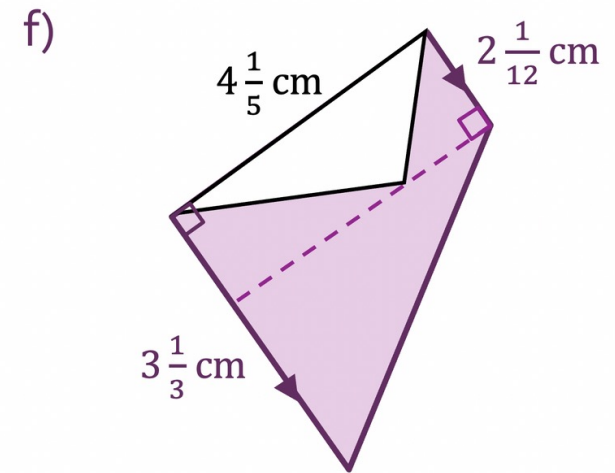
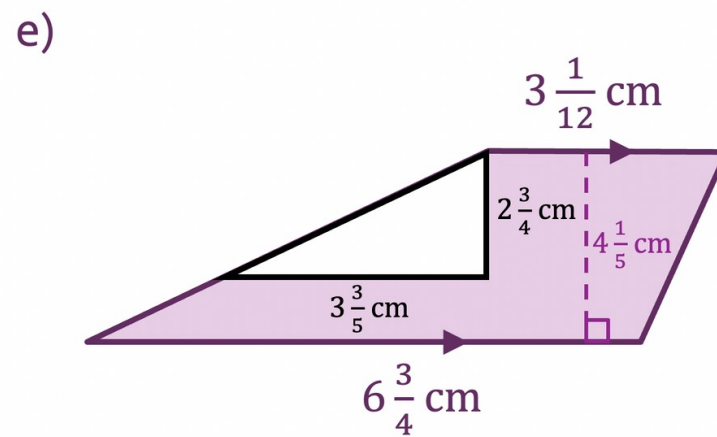
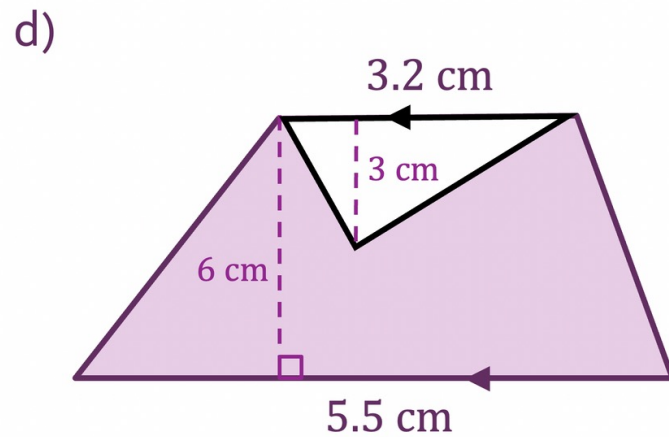
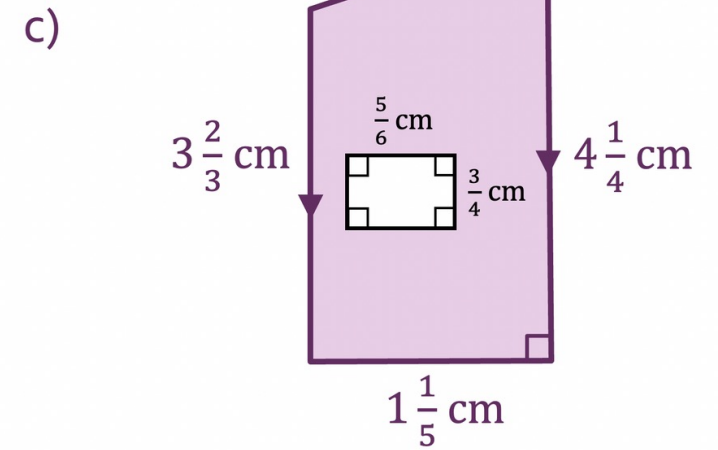
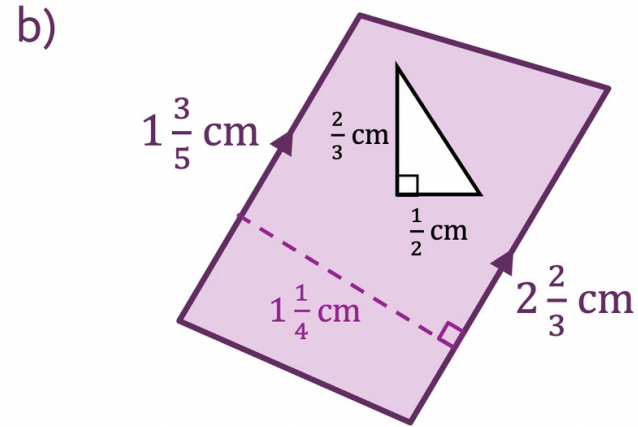
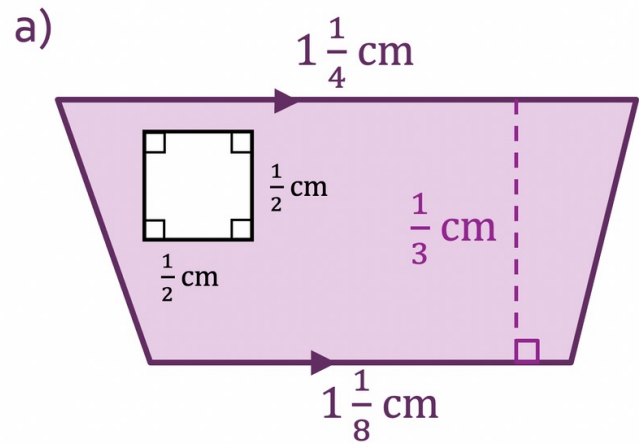
f)



From Karen Hancock

<https://interwovenmaths.com/areas-of-trapeziums>

Find each shaded area.



From Karen Hancock

<https://interwovenmaths.com/areas-of-trapeziums>

# How can we make questions that are more challenging?

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Require justifications	Generalise	Make things messy
Provide opportunities		



## A challenge, to be sure, but a welcome one

Using the digits from 1 to 9 exactly once each, fill in the gaps so that each equation has an integer solution.

$$\square(x - 3) = 49$$

$$4(x + 7) = \square$$

$$3(\square + x) = 12$$

$$6(x - 12) = \square x$$

$$10 = \square(x + 6)$$

$$9(x - 1) = \square$$

$$2(3x - 1) = \square$$

$$5(3 + \square x) = 25$$

$$92 = 4(11x + \square)$$

# How can we make questions that are more challenging?

Require justifications

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Provide opportunities

Allow for creativity...

# How can we make questions that are more challenging?

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Allow for creativity...

...but give restrictions

# Quintets

$$\square x + \square = \square x + \square \text{ has the solution } x = \square$$

Can you fill in the gaps with...

- a ... five different odd numbers?
- b ... five different even numbers?
- c ... five different prime numbers?
- d ... five different square numbers?
- e ... five consecutive numbers?  
(not necessarily in order)
- f ... five different factors of 32?
- g ... five different multiples of 7?

Can you fill the gaps so that each equation has a different solution?

# How can we make questions that are more challenging?

Require justifications

Generalise

Make things messy

Provide opportunities

Allow for creativity...

...but give restrictions

# How can we make challenging questions more accessible?

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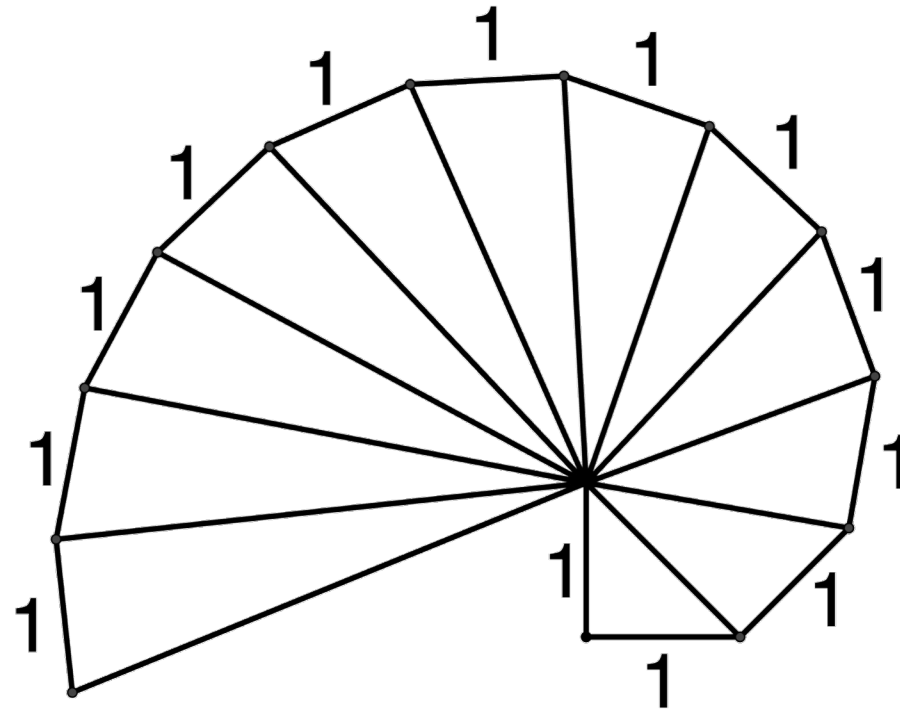
Require justifications	Generalise	<b>Scaffold</b>
Provide opportunities	Allow for creativity...	...but give restrictions

A rectangle has sides of length  $4y - x$ ,  $x + 4$ ,  $y + 10$ , and  $2y + 3$ .

How many different rectangles are possible?

# How many triangles could be added before it starts to overlap?

(All the triangles are right-angled.)





1. If each small square measured 1 cm by 1 cm, what would be the total area and perimeter of the flag?

a) Area =

b) Perimeter =

2. a) Approximate the area of the **orange** strip of the flag.

b) What shape is the **orange** stripe?

c) Which other stripe has the same area as **orange**? How do you know?

3. a) Explain why the top length of the **yellow** strip is exactly  $7\frac{1}{3}$  cm.

b) Calculate the area of **yellow**.

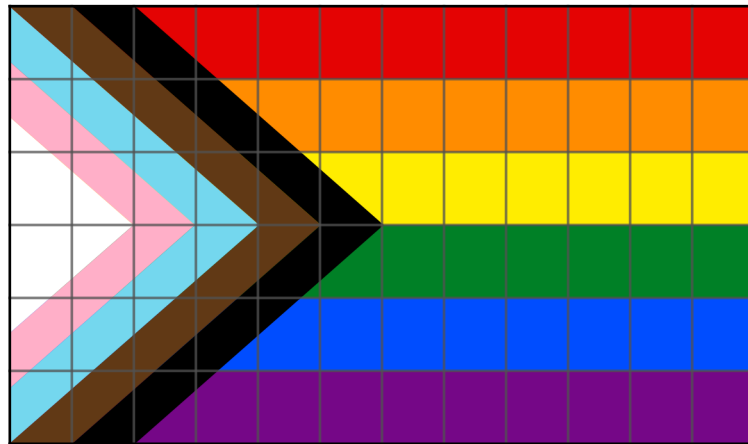
4. a) Use Pythagoras' Theorem to calculate the length of the diagonal side of the **green** strip.

b) Calculate the perimeter of **green**.

### Background Information

Shown to the right is the 'Progress Pride Flag' designed by Daniel Quasar in 2018.

It shows the traditional six colours of the rainbow pride flag used by LGBTQIA+ communities since the 1970s, with added triangular 'chevrons' representing additional groups such as transgender people and people of colour.



5. a) What are the gradients of the edges of the **black** chevron? Are any parallel? Perpendicular?

b) Calculate the length of each of those edges.

6. What percentage of the total area of the flag is **dark blue**?

7. If the origin was in the bottom-left corner of the flag, what would be the equation of the edge between **violet** and **black**?

8. a) What is the area of **white**?

b) What is the area of **pink**?

9. What fraction of the total area of the flag is **light blue**?

## Interwoven Maths



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### Product Rule for Counting Counting Factors

e.g. How many factors does 180 000 have?

$$180\,000 = 10\,000 \times 18$$

$$= 2^4 \times 5^4 \times 2 \times 3^2$$

$$= 2^5 \times 3^2 \times 5^4$$

Number of factors =  $(5+1)(2+1)(4+1) = 6 \times 3 \times 5 = 90$

- How many factors does 245 000 have?
- How many odd factors does 245 000?
- How many square numbers are factors of 245 000?
- How many factors of 245 000 are also factors of 180 000?
- How many factors of 180 000 are **not** also factors of 245 000?
- Find three other numbers that would each have the same number of factors as 245 000.

(IW) Product Rule for Counting Factors

### Introducing Sequences And Series an interwoven approach

(IW) Introducing Geometric Series

InterwovenMaths.com

#### Volume and Surface Area of a Cuboid

	A	B	C	Volume	Surface Area
a	1 cm	2 cm	10 cm		
b	1 cm	2 cm		10 cm <sup>3</sup>	
c	1 cm	2 cm			10 cm <sup>2</sup>
d	2 cm	5 cm	90 cm <sup>3</sup>		
e	2 cm	5 cm		90 cm <sup>3</sup>	
f	2 cm	5 cm			90 cm <sup>2</sup>
g	4 cm			400 cm <sup>3</sup>	400 cm <sup>2</sup>
h	4 cm			288 cm <sup>3</sup>	288 cm <sup>2</sup>
i	5 cm			250 cm <sup>3</sup>	250 cm <sup>2</sup>

(IW) Interwoven Volume and Surface Area Completion Tables

Prime numbers less than 100

Prime	Composite	Prime	Composite	Prime	Composite
2	4	11	16	22	28
3	6	13	18	23	29
5	10	17	20	29	35
7	14	19	24	31	36
11	22	23	28	37	42
13	26	29	34	41	46
17	34	31	38	43	48
19	38	37	44	47	52
23	46	41	48	49	54
29	56	43	50	53	58
31	60	47	54	59	64
37	68	53	60	61	68
41	74	59	66	67	72
43	78	61	70	71	76
47	84	67	74	73	80
53	90	71	78	79	84
59	96	73	82	83	88
61	100	79	86	89	92
67		83	90	97	98
71		89	94		
73		97	96		
79					
83					
89					
97					

(IW) Histograms and Cumulative Frequency

#### Fractions

Find the mean, median, and range of:  $3\frac{1}{2}$ ,  $6\frac{1}{6}$ , and  $2\frac{1}{2}$

#### Area and Perimeter

A rectangle has a width of 3 cm and a height of 2 cm. Draw a second rectangle so that the two rectangles have a mean area of 13 cm<sup>2</sup> and have perimeters with a range of 8 cm.

(IW) Interwoven Revision Questions to Project

#### Standard Form

Find the median of the following:  $3 \times 10^{-4}$ ,  $4 \times 10^{-3}$ ,  $5 \times 10^{-6}$ ,  $6 \times 10^{-5}$ .

#### Averages with...

Find the upper bound for the median angle in a quadrilateral. Is it possible to actually draw a quadrilateral with that median angle?

#### Angles in polygons with...

**Mean**  
A regular polygon has interior and exterior angles in the ratio 5:1.  
How many sides does it have?

**Perimeter**  
A regular polygon has exterior angles that are 2 1/2% of the size of the angle of its interior angles.  
How many sides does it have?

**Area**  
A regular polygon has interior angles that round to 100° to 3 significant figures.  
How many sides could it have?

(IW) Angles in Polygons with... (Mixed Questions)

#### Circle Theorems

Circle Theorems  
In the circle below, all other points are on the circumference. All angles are marked with the letter A.  
 $\angle ADB = 20^\circ$   
What is  $\angle ACB$ ?

(IW) Circle Theorems with... (no diagrams!)

#### Circle Theorems

Circle Theorems  
In the circle below, all other points are on the circumference. All angles are marked with the letter A.  
 $\angle ABC = 20^\circ$   
What is  $\angle ACB$ ?

#### Trigonometry with...

**Area**  
Find the area of the triangle below.

**Angles in Triangles**  
Find the value of x.

**Pythagoras**  
Find the length of the hypotenuse.

#### Pythagoras with...

**Area and Perimeter**  
Find the area and perimeter of the shape below.

**Standard Form**  
Find the value of x.

**Angles**  
Find the value of x.

#### Averages with...

**(1)** Which question is the odd one out? Why?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
Can the other questions be adapted in the same way?

**(2)** How could you change that question so that it would fit with the others?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
How many miles would she work at 28 hours?"

**(3)** What does 3 represent in each question?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"

**(4)** How could the graph be used to represent each of the other questions?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"

**(5)** What extra questions could be added that would fit with the others?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"

**(6)** What does 1.5 represent in each question?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"

**(7)** How could a ratio table be used for each question?  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"  
"Myra worked 4 miles at 2.2 hours. How many miles would she work at 28 hours?"

#### Areas of Trapeziums with... Fractions and Decimals

Find the area of each trapezium.

a)  $3.5\text{ cm}$ ,  $2.8\text{ cm}$ ,  $4\text{ cm}$

b)  $6\text{ cm}$ ,  $3.4\text{ cm}$ ,  $3.8\text{ cm}$

c)  $2.5\text{ cm}$ ,  $2\text{ cm}$ ,  $1\text{ cm}$

d)  $5.2\text{ cm}$ ,  $4.8\text{ cm}$ ,  $1.1\text{ cm}$

e)  $4.1\text{ cm}$ ,  $1\text{ cm}$ ,  $1\text{ cm}$

# Thank you!



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📅 Joined March 2015

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